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WIENER INDEX OF ROUGH CO-ZERO DIVISOR GRAPH OF A ROUGH SEMIRING

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ABSTRACT. In this proposed article we consider an approximation space $I = (U, R)$, where U denotes nonempty finite set of objects and R be an arbitrary equivalence relation defined on U. The Rough Co-zero divisor graph $G(Z^*(J))$ of a Rough Semiring (T, Δ, ∇) on I corresponding to the Rough ideal is taken for study. The degree of each of the vertices and distance of any two vertices in $G(Z^*(J))$ are computed. Based on the degree of vertices a Partition graph $P(Z^*(J))$ is defined. This Partition graph is used to find the Wiener index of $G(Z^*(J))$. The main advantage of partition graph is that all the graph theoretical parameters can be computed for any Rough Co-zero divisor graph with $2^{n-m}.3^m - 2, 1 \leq m \leq n$. An analysis of disease symptom relationship is made through the defined parameters. All of the concepts are embellished with suitable examples. AMS Classification: 05C12, 05C05

Keywords: Degree, Distance, Rough Co-Zero Divisor Graph, Partition Graph, Wiener Index.

1.INTRODUCTION

Wiener index is the primary graph theoretical index to be used in Chemistry. It was introduced by Harold Wiener in 1947. Mathematical studies of Wiener index started in the year 1970 [1];[2]. The concept of Wiener index is a boundary between Algebraic graph theory and Chemistry.

The introduction of Rough set theory was proposed by Pawlak [12] in 1982 which is tool for Medalling with imperfect knowledge, in particular with vague concepts in the information systems and it is defined as a pair called lower and upper approximation.

Rough set theory is one such tool and has more advantage than Fuzzy set theory and any other theory like probability theory, evidence theory, etc and It provides efficient methods, algorithms and tools for finding hidden patterns in data. For a subset of the universe, Rough set is an ordered pair of lower and upper approximations. The research on Rough set theory and its application in various fields have attracted the attention of researchers more and more. Rough set theory is applied in knowledge discovery, feature selection, pattern recognition, machine learning, medicine, and telecommunications.

In 201[5];[8-10] considered an approximation space $I = (U, R)$ where U is nonempty finite set of objects and R is an arbitrary equivalence relation on U. With respect to the two operations Δ and ∇ the set of all Rough sets T on U is proved to be a Semiring called namely Rough Semiring. The ideals of this Rough Semiring are also studied widely [5].

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Hua [6];[7] designed the concept of Co-zero divisor graph, denoted by $\Gamma'(R)$, on a commutative ring R. Let by $W^*(R)$ denote non-unit elements of R. In the vertex set $\Gamma'(R)$ is $W^*(R)$ and for two distinct vertices a and b in $W^*(R)$, a is adjacent to b if and only if $a\epsilon(b)$ and $b\epsilon(a)$, where (c) is an ideal generated by the element c.

In 1988 Mohar and Pisanki they derived algorithms to calculate the Wiener index of graphs and trees. In 2013 Vijayabarathi and Anjaneyulu also have made a study on the Wiener index of graphs and its chemical applications. M. Fischerman, et.al (characterized the trees which minimizes and maximizes the Wiener index under different conditions and Stevanovic discussed on the maximization of Wiener index of graphs with maximum degree. A study on edge - Wiener index of a graph is made by authors P. Danklemann et.al.[3]; [4] Motivation of this study is to discuss the Algebraic graph theoretic concepts on the Rough semiring (T, Δ, ∇). The complexity of this study will increase for large values of n and m. In this work, the complexity is made simpler by defining a partition graph $P\big(Z^*(J)\big)$corresponding to $G(Z^*(J))$, in which the vertices are divided into seven partitions. This partition graph is obtained by defining suitable partition in the vertices of $G(Z^*(J))$. Hence vertices of same degree will fall into same partition. The objective of these graph theoretical parameters is computed using partition graph corresponding to a Rough Co-zero divisor graph.

In section 2 we provide with a basic definitions and notations and in section 3 we define the partition graph of $G(Z^*(J))$ and in section 3.1 we find the degree's of the vertices in $G(Z^*(J))$. In section 3.2. we define partition graph of Rough Co-zero divisor graph. Finally section 3.3 explains the Wiener index of $G(Z^*(J))$ using the partition graph. An analysis of disease symptom relationship is made through the defined parameters in section 4 followed by a conclusion.

2 . BASIC DEFINITION AND NOTATION

2.1 Graph Theory

Graph theory plays a vital role in modelling real time problems with which a suitable solution can be obtained. The Graph theoretical parameters are widely used in data flow diagram, decision making ability, and displays relationships among objects, easy alterations and modifications in the existing system etc.

Let graph S, connected undirected graph with $V(S) = \{v_1, v_2, \dots v_n\}$ and $E(S) = \{e_1, e_2, \dots e_n\}$ the distance between any two vertices v_i and v_j denoted by $d(v_i, v_j)$ is the length of a shortest path between v_i and v_j in S. The wiener index of a graph S denoted by $W(S)$ is the sum of the distances between all pair of vertices of S.

(i.e.)$W(S) = \sum_{i<j} d(v_i, v_j)$

The degree of a vertex of a graph is the number of edges that are incident to the vertex. The degree of a vertex is denoted by $deg(v)$.

2.2 Rough Set Theory

Let U be a nonempty set which is finite and R be an arbitrary equivalence relation defined on Uthen $I = (U, R)$ is called an approximation space for x, $[x]_R = \{y \in U | (x, y) \in R\}$ is said to be an equivalence class. Then for $X \subseteq U$, let $RS(X) = \left(\underline{R}(X), \overline{R}(X)\right)$ be the rough set, where $\underline{R}(X) = \{x \in U | [x]_R \subseteq X\}$ is said to be a lower approximation and higher approximation defined as $\overline{R}(X) = \{x \in U | [x]_R \cap X \neq \emptyset\}$. Also $T = \{RS(X) | X \subseteq U\}$ be the collection of all rough sets.

For any approximation space I = (U, R), the set of all Rough sets T was proved to be a lattice called Rough lattice having prabaΔ and praba∇ as its least upper bound and greatest lower bound. Hence (T,Δ,∇) is a semiring called rough semiring [9].

Theorem 2.1.

Let I = (U, A) be an information system where U be the universal (finite) set and A be the set of attributes and T be the set of all rough sets then (T, Δ, ∇) is a Semiring.

Definition 2.1

Let $X, Y \subseteq U$ the $praba\,\Delta$ is defined as $X = X \cup Y$ if $IW(X \cup Y) = IW(X) + IW(Y) - IW(X \cap Y)$.

Definition 2.2:

Let $X, Y \subseteq U$ then an element $x \in U$ is called a Pivot element, if $[x]_p \nsubseteq X \cap Y$, but $[x]_p \cap X \neq \emptyset$ and $[x]_p \cap Y \neq \emptyset$. $P_{X \cap Y}$ be the collection of pivot elements.

Definition 2.3:

$praba\,\nabla$ X and Y define by $X \nabla Y = \left\{x | [x]_p \subseteq X \cap Y\right\} \cup P_{X \cap Y}$, where $X, Y \subseteq U$. Identify that every pivot element in $P_{X \cap Y}$ be the representative of specific class.

Definition 2.4:

The Rough co-zero divisor graph $G\left(Z^*(J)\right) = \left(V\left(Z^*(J)\right), E\left(Z^*(J)\right)\right)$ where $V\left(Z^*(J)\right)$ is the set of vertices consisting of the elements of $T^* = T - \{RS(\emptyset), RS(U)\}$ and two elements $RS(X)$, $RS(Y) \in V\left(Z^*(J)\right)$ are adjacent if and only if $RS(X) \notin RS(Y)\nabla J$ and $RS(Y) \notin RS(X)\nabla J$.

Illustration 2.1:

Let $U = \{x_1, x_2, x_3, x_4, x_5, x_6\}$ and let $\{X_1, X_2, X_3\}$ are the equivalence classes induced by an equivalence relation R on U such $X_1 = \{x_1, x_3\}$, $X_1 = \{x_2, x_4, x_6\}$ and $X_1 = \{x_5\}$

$V(Z^*(J)) = \{RS(x_1), RS(x_2), RS(X_1), RS(X_2), RS(X_3), RS(x_1 \cup x_2), RS(X_1 \cup X_2), RS(X_1 \cup X_3), RS(X_2 \cup X_3), RS(x_1 \cup X_2), RS(X_1 \cup x_2), RS(x_1 \cup X_3), RS(x_2 \cup X_3), RS(x_1 \cup X_2 \cup X_3), RS(X_1 \cup x_2 \cup X_3), RS(x_1 \cup x_2 \cup X_3)\}$; $B = \{x_1, x_2\}, J = \{RS(x_1), RS(x_2), RS(x_1 \cup x_2)\}$

Figure 1 represents the Rough co-zero divisor graph for $n = 3$ and $m = 2$.

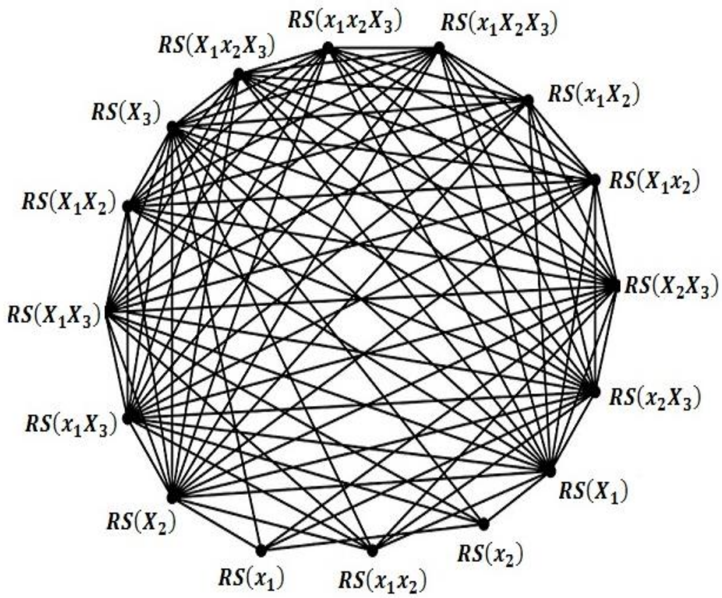

Figure 1: Rough Co-Zero Divisor Graph for n = 3 and m = 2.

Illustration 2.2:

Let $U = \{x_1, x_2, x_3, x_4, x_5, x_6\}$ and let $\{X_1, X_2, X_3\}$ are the equivalence classes induced by an equivalence relation R on U such $X_1 = \{x_1, x_3\}$, $X_1 = \{x_2, x_4\}$ and $X_1 = \{x_5, x_6\}$

$V(Z^*(J)) = \{RS(x_1), RS(x_2), RS(x_3), RS(X_1), RS(X_2), RS(X_3), RS(x_1 \cup x_2), RS(x_1 \cup x_3), RS(x_2 \cup x_3), RS(x_1 \cup x_2 \cup x_3), RS(X_1 \cup X_2), RS(X_1 \cup X_3), RS(X_2 \cup X_3), RS(x_1 \cup X_2), RS(X_1 \cup x_2), RS(x_1 \cup X_3), RS(X_1 \cup x_3), RS(x_2 \cup X_3), RS(X_2 \cup x_3), RS(x_1 \cup X_2 \cup X_3), RS(X_1 \cup x_2 \cup X_3), RS(X_1 \cup X_2 \cup x_3), RS(x_1 \cup x_2 \cup X_3), RS(x_1 \cup X_2 \cup x_3), RS(X_1 \cup x_2 \cup x_3)\}$

$B = \{x_1, x_2, x_3\}$, $J = \{RS(x_1), RS(x_2), RS(x_3), RS(x_1 \cup x_2), RS(x_1 \cup x_3), RS(x_2 \cup x_3) RS(x_1 \cup x_2 \cup x_3)\}$

Figure 2 represents the Rough co-zero divisor graph for $= m = 3$.

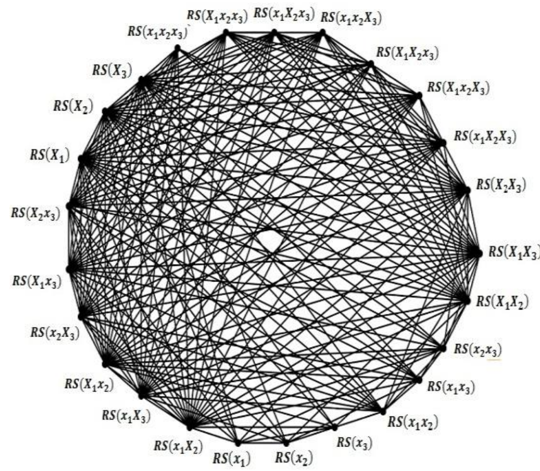

Figure 2: Rough Co-Zero Divisor Graph for n = m = 3

3. WIENER INDEX OF A ROUGH CO-ZERO DIVISOR GRAPH

Throughout this section we assume that $I = (U, R)$. Let $\{X_1, X_2, .. X_n\}$ are the equivalence classes persuaded by R on U .We also infer that m equivalence classes $\{X_1, X_2, .. X_m\}$ with cardinality larger than 1and the remaining $n - m$ equivalence classes $\{X_{m+1}, X_{m+2}, .. X_n\}$ have cardinality equal to 1, where $1 < m \leq n$. Let $B = \{x_1, x_2, .. x_m\}$ representative member of the equivalence class having cardinality larger than 1.

Let $T = \{RS(X) | X \subseteq U\}$ be the rough lattice [9] associated with the information system I and $T^* = T - \{RS(\emptyset), RS(U)\}$. Moreover let $J = \{RS(X) | X \in P(B)\}$ be the rough ideal [5] on T^*.

3.1 Degree of Rough Co-Zero Divisor Graph

In this section we obtain the degree's each vertex of $G\big(Z^*(J)\big)$.

Definition 3.1.1:

Degree of a vertex in $G\big(Z^*(J)\big)$ is denoted by $deg(RS(X))$, is the no. of lines incident with $RS(X)$ .The maximum degree of $G\big(Z^*(J)\big)$ denoted by $\Delta(G(Z^*(J)))$, is defined to be $\Delta(G(Z^*(J))) = max\big\{deg\big(RS(X)\big) | RS(X) \in V\big(Z^*(J)\big)\big\}$.

Similarly the minimum degree of $G\big(Z^*(J)\big)$ denoted by $\Delta(G(Z^*(J)))$, is defined to be $\delta(G(Z^*(J))) = min\big\{deg\big(RS(X)\big) | RS(X) \in V\big(Z^*(J)\big)\big\}$.

Theorem 3.1.1

The degree of $RS(x_i)$ is $(m-1)(2^{n+1-m}) + 2^{m-1} - m + 2^{n-m} - 1 + [(m-1)C_r(2^{n-m} - 1) + (m-1)C_r(2^{n-m}) + \sum_{r=2}^{m-1}\big((m-1)C_1 + (m-1)C_2 + \cdots + (m-1)C_{r-1}\big)](2^{n-m})$

Proof:

Let us consider the Rough set corresponding to the pivot elements. Let $P_1 = \{RS(x_i) | i = 1,2..m\}$ the following observations are made.

1. For each i, $RS(x_i)$ is connected to all $RS(x_j)$ for $i \neq j$, $i,j = 1,2 \ldots m$. Number of such elements to which $RS(x_i)$ connected to is $(m-1)$.
2. For each i, $RS(x_i)$ is connected to $RS(x_j \cup M') \cup RS(X_j \cup M)$ for $i \neq j$, $i,j = 1,2 \ldots m$. Number of such elements to which $RS(x_i)$ connected to is $(m-1)(2^{n-m+1}-1)$.
3. For each i, $RS(x_i)$ is connected to all the elements of the set $\{RS(Q_j) | Q_j \in P(Q) - \emptyset, Q = X_{m+1}, X_{m+2,\ldots} X_n\}$ for $i \neq j$, $i,j = 1,2 \ldots m$. Number of such elements to which $RS(x_i)$ connected to is $2^{n-m}-1$.
4. We consider the set $\{RS(x_1, x_2 \ldots x_r) | 1 < r < m\}$, $RS(x_i)$ will be connected to all the elements of the form $RS(x_1, x_2 \ldots x_r)$ that does not contain $RS(x_i)$. Number of such elements is given by $2^{m-1}-m$.
5. We consider the set $RS(x_1, x_2 \ldots x_r \cup M') \cup RS(X_1, X_2 \ldots X_r \cup M) \cup RS(Q_r \cup M)$. Number of such elements to which $RS(x_i)$ connected to is $(m-1)C_r(2^{n-m}-1) + (m-1)C_r(2^{n-m}) + \sum_{r=2}^{m-1}\left((m-1)C_1 + (m-1)C_2 + \cdots + (m-1)C_{r-1}\right)(2^{n-m})$

   Therefore for the total number of elements in $V(Z^*(J))$ to which $RS(x_i)$ is connected to is obtained by adding all the observations. Hence degree of $RS(x_i)$ is

   $(m-1)(2^{n-m+1}) + 2^{n-m} - 1 + 2^{m-1} - m + \left[(m-1)C_r(2^{n-m}-1) + (m-1)C_r(2^{n-m}) + (2^{n-m})\sum_{r=2}^{m-1}\left((m-1)C_1 + (m-1)C_2 + \cdots + (m-1)C_{r-1}\right)\right]$

Theorem 3.1.2:

The degree of $RS(x_i \cup M') \cup RS(X_i \cup M)$ is $m(2^{n+1-m}-1) + 2^m - m - 2 + 2^{n-m} - 1 + 1 + 2^m - (m+2)(2^{n-m+1}-1) + 3(3^{m-1} - 2^m + 1)(2^{n-m}) + 2^{n-m}(2^m) - 2$

Where M is the union of none, one or more equivalence classes whose cardinality is equal to one in $G(Z^*(J))$ and M' denotes the one either more equivalence classes having cardinality is equivalent to one in $G(Z^*(J))$.

Proof:

Now let us consider $P_2 = \{RS(x_i \cup M') \cup RS(X_i \cup M) | i = 1,2..m\}$

To calculate the degree of vertices in this set, consider the following statements.

1. For each i, $RS(x_i \cup M') \cup RS(X_i \cup M)$ will not connected to $RS(x_i)$. Hence number of such elements to which $RS(x_i)$ connected to is $(m-1)(2^{n-m+1}-1)$.
2. For each i, $RS(x_i \cup M')$ is connected to all $RS(x_i \cup M') \cup RS(X_i \cup M)$ and $RS(X_i \cup M)$ is connected to all $RS(x_i \cup M') \cup RS(X_i \cup M)$. Therefore the total sum of such elements to which $RS(x_i \cup M') \cup RS(X_i \cup M)$ connected to is $(2^{n-m+1}-1)$.

3. Evidently $RS(x_i \cup M') \cup RS(X_i \cup M)$ is connected to all the elements of the set $\{RS(Q_j)|Q_j \in P(Q) - \emptyset, Q = X_{m+1}, X_{m+2,\dots}X_n\}$. Number of such elements to which $RS(x_i \cup M') \cup RS(X_i \cup M)$ connected to is $2^{n-m} - 1$.
4. For each i, every elements in $RS(x_i \cup M') \cup RS(X_i \cup M)$ is connected to the set $\{RS(x_1, x_2 \dots x_r)|1 < r < m\}$. Number of such elements is given by $2^m - m - 2$.
5. Similarly for each i, the set P_2 is connected to the single element set $RS(x_1, x_2 \dots x_m)$. Hence the number of such element is 1.
6. For each i, every elements in P_2 is connected to the set $RS(x_1, x_2 \dots x_r \cup M')$ with $(2^m - (m+2))(2^{n-m})$ elements and P_2 is connected to the set $RS(X_1, X_2 \dots X_r \cup M)$ with $(2^m - (m+2))(2^{n-m} - 1)$ elements and P_2 is connected to the set $RS(Q_r \cup M)$ where $Q_r = x_r$ or X_r with $3(2^{n-m})(3^{m-1} - 2^m + 1)$ elements.
7. For each i, every elements in P_2 is connected all the elements in the set $RS(x_1, x_2 \dots x_m \cup M') \cup RS(X_1, X_2 \dots X_m \cup M) \cup RS(Q_m \cup M)$with $2^{n-m}(2^m) - 2$ elements.
8. Therefore for the total number of elements in $V(Z^*(J))$ to which $RS(x_i \cup M') \cup RS(X_i \cup M)$ is connected to is obtained by adding all the statements. Hence degree of $RS(x_i \cup M') \cup RS(X_i \cup M)$ is
$m(2^{n+1-m} - 1) + 2^m - m - 2 + 2^{n-m} - 1 + 1 + 2^m - (m+2)(2^{n-m+1} - 1) + 3(3^{m-1} - 2^m + 1)(2^{n-m}) + 2^{n-m}(2^m) - 2$

Theorem 3.1.3

The degree of the elements in the set $\{RS(Y)|Y \in M'\}$ for $i = 1,2,..m$ is $(2^m + 1)2^{n-m} - (m+2)(2^{n+1-m} - 1) + 2^m - m - 2 + 2^m + 3(2^{n-m})(3^{m-1} - 2^m + 1) + 2^{n-m}(2^m) - 2$

Proof:

Let $P_3 = \{RS(Y)|Y \in M'\}$. A similar argument as in Theorem 3.1.2 is true for the elements of the set $\{RS(Y)|Y \in M'\}$ and it is connected to all the elements in $V(Z^*(J))$. Hence the total number of such elements in P_3 is $(2^m + 1)2^{n-m} - (m+2)(2^{n+1-m} - 1) + 2^m - m - 2 + 2^m + 3(2^{n-m})(3^{m-1} - 2^m + 1) + 2^{n-m}(2^m) - 2$.

Accurately the degree of each vertices in the set $\{RS(Y)|Y \in M'\}$ is $2^{n-m}.3^m - 3$.

Theorem 3.1.4

The degree of elements in $RS(x_1, x_2 \dots x_r)$ is $2^{n-m}(2^m) - 2^{m-1} - 2 + \left\{\left(\frac{m(m-1)}{2} - 1\right) + \left(\frac{m(m-1)(m-2)}{2} - 1.2 + 2.3 + \cdots (m-2)(m-1)\right) + \cdots + 1 - 2 + \left(|A| - (* + ** + ***)\right)\right\}$

Where $|A| = 2^{n-m}(2^m) - 2$

$$*=\left\{\left[\frac{m(m-1)}{2}\right].[(m-2)C_0+(m-2)C_1+\cdots+(m-2)C_{m-3}]+\left[\frac{m(m-1)(m-2)}{2}-1.2+2.3+\cdots(m-2)(m-1)\right].[(m-3)C_0+(m-3)C_1+\cdots+(m-3)C_{m-4}]+\cdots+1.\left[m\big(m(m-1)\big).(2^{n-m}-1)\right]\right\}$$

$$**=\left\{\left[\frac{m(m-1)}{2}\right].[(m-2)C_0+(m-2)C_1+\cdots+(m-2)C_{m-3}]+\left[\frac{m(m-1)(m-2)}{2}-1.2+2.3+\cdots(m-2)(m-1)\right].[(m-3)C_0+(m-3)C_1+\cdots+(m-3)C_{m-4}]+\cdots+1.\left[m\big(m(m-1)\big).(2^{n-m})\right]\right\}$$

$$***=\left\{\left[\frac{m(m-1)}{2}\right]\left[\sum_{k=2}^{m-1}\sum_{i=1}^{k-1}kC_i\right]+\left[\frac{m(m-1)(m-2)}{2}-1.2+2.3+\cdots(m-2)(m-1)\right]+\left[\sum_{k=3}^{m-1}\sum_{i=1}^{k-1}kC_i\right]+..+1\right\}2^{n-m}$$

Proof:

Consider the set $P_4=RS(x_1,x_2\ldots x_r)$ where $1<r<m$.

When $r=2$, An element $RS(Z)$ will be connected to $RS(x_1,x_2)$ precisely if $RS(Z)\notin RS(x_1,x_2)\nabla J$ and $RS(x_1,x_2)\notin RS(Z)\nabla$J. The degree of $RS(x_1,x_2)$ is acquired by considering those vertices of $RS(Z)$ connected to $RS(x_1,x_2)$. Hence $RS(x_1,x_2)$ is connected to P_1 with $(m-2)$ elements where $i\neq 1,2$ and it is connected to P_2 with $2m(2^{n-m})-m$ elements and $RS(x_1,x_2)$ is connected to P_3 with $2^{n-m}-1$ elements.

When $r=3$, $RS(x_1,x_2,x_3)$is connected to P_1 with $(m-2)$ elements where $i\neq 1,2,3$ and it is connected to P_2 with $2m(2^{n-m})-m$ elements and $RS(x_1,x_2,x_3)$ is connected to P_3 with $2^{n-m}-1$ elements. Clearly it is true for $r=m-1$.

The degree of elements in P_4 is $2^{n-m}(2^m)-2^{m-1}-2+\left\{\left(\frac{m(m-1)}{2}-1\right)+\left(\frac{m(m-1)(m-2)}{2}-1.2+2.3+\cdots(m-2)(m-1)\right)+\cdots+1-2+\big(|A|-(*+**+***)\big)\right\}$

Theorem 3.1.5:

The degree of $RS(x_1,x_2,\ldots x_m)$ is $2^{n-m}[2m+2^m-(m+2)+3(3^{m-1}-2^m+1)]-m$

Proof:

Now we consider the set $P_5=RS(x_1,x_2,\ldots x_m)$ for every $x_i\in B$. A vertex $RS(X)$ will be connected to $RS(x_i)$ for $i=1,2,\ldots m$ if and only if $RS(X)\notin RS(x_i)\nabla J$ and $RS(x_i)\notin RS(X)\nabla$J. The degree of P_5 (single element) is achieved by taking into account of those vertices to which $RS(X)$ which are adjacent to $RS(x_1,x_2,\ldots x_m)$. Also $RS(x_1,x_2,\ldots x_m)$ is not connected to the set

$P(B)$, where $B = \{x_1, x_2, \dots x_m\}$. Hence the degree of each elements in the set $RS(x_1, x_2, \dots x_m)$ is $2^{n-m}[2m + 2^m - (m + 2) + 3(3^{m-1} - 2^m + 1)] - m$.

Theorem 3.1.6:

The degree of $\{RS(x_1, x_2, \dots x_r \cup M')|1 < r < m\}$ is $2^{n-m}[2m + 2^m + 1 + 3(3^{m-1} - 2^m + 1)] + [2^m - (m + 2)](2^{n-m+1} - 1) - 8$

Proof:

Let us consider $\{RS(x_1, x_2, \dots x_r \cup M')|1 < r < m\}$.

Observe that the set $\{RS(x_1, x_2, \dots x_r \cup M')|1 < r < m\}$ will not be connected to the elements in this set $(x_1, x_2, \dots x_r)$. Also $RS(x_1, x_2, \dots x_r \cup M') \notin RS(x_i \cup M') \cup RS(X_i \cup M)\nabla$J and $RS(x_i \cup M') \cup RS(X_i \cup M)) \notin RS(x_1, x_2, \dots x_r \cup M')\nabla$J.

Hence it is adjacent to all the elements of the set P_3 and P_5. Therefore the degree of $\{RS(x_1, x_2, \dots x_r \cup M')|1 < r < m\}$ is $2^{n-m}[2m + 2^m + 1 + 3(3^{m-1} - 2^m + 1)] + [2^m - (m + 2)](2^{n-m+1} - 1) - 8$

Corollary 1:

The degree of $\{RS(X_1, X_2, \dots X_r \cup M)|1 < r < m\}$ is $2^{n-m}[2m + 2^m + 1 + 3(3^{m-1} - 2^m + 1)] + [2^m - (m + 2)](2^{n-m+1} - 1) - 8$

Proof:

The proof is follows from Theorem 3.1.6. Consider $\{RS(X_1, X_2, \dots X_r \cup M)|1 < r < m\}$ where $|X_i| > 1$

Hence the degree of $\{RS(X_1, X_2, \dots X_r \cup M)|1 < r < m\}$is $2^{n-m}[2m + 2^m + 1 + 3(3^{m-1} - 2^m + 1)] + [2^m - (m + 2)](2^{n-m+1} - 1) - 8$

Corollary 2:

The degree of $RS(Q_r \cup M)$ is $2^{n-m}[2m + 2^m + 1 + 3(3^{m-1} - 2^m + 1)] + [2^m - (m + 2)](2^{n-m+1} - 1) - 8$

Proof:

The proof is similar to Theorem 3.1.6. The degree of $RS(Q_r \cup M), Q_r = \{(Z_1, Z_2, \dots Z_r)|Z_i = y_i \text{ } or \text{ } Y_i\}$ and $i = 1,2, \dots m$ is $2^{n-m}[2m + 2^m + 1 + 3(3^{m-1} - 2^m + 1)] + [2^m - (m + 2)](2^{n-m+1} - 1) - 8$

Theorem 3.1.7:

The degree of $RS(x_1, x_2, \dots x_m \cup M')$ is $2^{n-m}[2^{m+1} + m - 2 + 3(3^{m-1} - 2^m + 1)] - (m + 3)$

Proof:

Let us take the set $RS(x_1, x_2, \dots x_m \cup M')$ note that $RS(x_1, x_2, \dots x_m \cup M')$ is not connected to the elements of $P(B)$. But $RS(x_1, x_2, \dots x_m \cup M')$ is connected to the elements in $RS(x_1, x_2, \dots x_r \cup M') \cup RS(X_1, X_2, \dots X_r \cup M) \cup RS(Q_r \cup M)$ where $1 < r < m$ and also it is connected to P_2 and P_3. Hence the degree of $RS(x_1, x_2, \dots x_m \cup M')$ is $2^{n-m}[2^{m+1} + m - 2 + 3(3^{m-1} - 2^m + 1)] - (m + 3)$

Corollary 3:

The degree of $RS(X_1, X_2, \dots X_m \cup M)$is $2^{n-m}[2^{m+1} + m - 2 + 3(3^{m-1} - 2^m + 1)] - (m + 3)$

Proof:

The proof is obvious from Theorem 3.1.7.

Corollary 2:

The degree of $RS(Q_m \cup M)$is $2^{n-m}[2^{m+1} + m - 2 + 3(3^{m-1} - 2^m + 1)] - (m + 3)$

Proof:

The proof is similar to Theorem 3.1.7.

3.2. Partition Graph

In this section we define a partition on $V(Z^*(J))$. A partition graph is defined using this partition which is based on the degree's of vertices in $G(Z^*(J))$. The objective of this partition graph is to make the study of $G(Z^*(J))$ simpler. Because all the vertices in one partition will have the same degree and they behave similarly.

Definition 3.2.1:

The partition graph $P(Z^*(J))$ is a graph whose vertices are the partitions on $V(Z^*(J))$. Hence the vertices of $P(Z^*(J))$ is the set $\{P_1, P_2, P_3, P_4, P_5, P_6, P_7\}$, where

$P_1 = RS(x_i)$

$P_2 = RS(x_i \cup M') \cup RS(X_i \cup M)$

$P_3 = \{RS(Y) | Y \in M'\}$

$P_4 = RS(x_1, x_2, \dots x_r)$

$P_5 = RS(x_1, x_2, \dots x_m)$

$P_6 = RS(x_1, x_2, \dots x_r \cup M') \cup RS(X_1, X_2, \dots X_r \cup M) \cup RS(Q_r \cup M)$

$P_7 = RS(x_1, x_2, \dots x_m \cup M') \cup RS(X_1, X_2, \dots X_m \cup M) \cup RS(Q_m \cup M)$

Two vertices P_i and P_j in the partition graph are connected by an edge if the elements in P_i are adjacent to any of the elements in P_j by an edge in $G(Z^*(J))$.

Note:

It is very important to notice that the partition graph of $G(Z^*(J))$ always has 7 vertices and the number of elements in each of the partition will vary as m and n varies. Also note that when $n = m, M' = \{RS(Y) | Y \in P(X_{m+1}, X_{m+2}, \dots X_n)\} = \emptyset$, therefore $P_3 = \emptyset$. Hence when $n = m$, the number of vertices in the partition graph of $G(Z^*(J))$ has only 6 vertices.

The elements of P_i for all i, $i \neq 4$ will form a complete graph. When $i = 4$, the elements of P_4 will form a complete graph for $m = 3$ and for all n. It is easy to verify that it need not be true for $m > 3$.

The following figure 3 represents the partition graph of $G(Z^*(J))$ for $n \neq m$

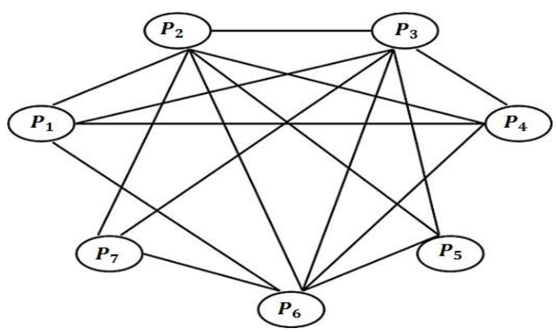

Figure 3: Partition Graph for n ≠ m

When $n = m$ the corresponding partition graph of $G(Z^*(J))$ is given in figure 4

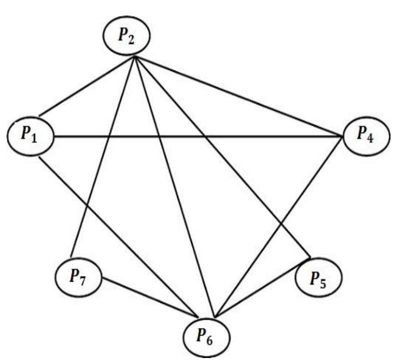

Figure 4: Partition Graph for n = m

Now the following table gives the elements of partition graph and the cardinality of each partition for all values of m and n.

Table 1: Partitions and Cardinalities of $G(Z^*(J))$

Partitions (P_i)	**Elements**	**Cardinality**
P_1	$RS(x_i)$	m
P_2	$RS(x_i \cup M') \cup RS(X_i \cup M)$	$2m(2^{n-m}) - m$
P_3	$\{RS(Y) \mid Y \in M'\}$	$2^{n-m} - 1$
P_4	$RS(x_1, x_2, \dots x_r)$	$2^m - (m+2)$
P_5	$RS(x_1, x_2, \dots x_m)$	1
P_6	$RS(x_1, x_2, \dots x_r \cup M') \cup RS(X_1, X_2, \dots X_r \cup M) \cup RS(Q_r \cup M)$	$2^m - (m+2)(2^{n-m+1} - 1$ $+ 3(2^{n-m})(3^{m-1} - 2^m + 1)$
P_7	$RS(x_1, x_2, \dots x_m \cup M') \cup RS(X_1, X_2, \dots X_m \cup M) \cup RS(Q_m \cup M)$	$2^{n-m}(2^m) - 2$

Examples

Example 3.2.1

From Illustration 1 in section 2, we obtain the elements in each partition of $G(Z^*(J))$ using table 1.

Table 2: Partitions and Cardinalities of $G(Z^*(J))$ for $n = 3$ & $m = 2$

Partitions (P_i)	**Elements**	**Cardinality**
P_1	$RS(x_1), RS(x_2)$	2
P_2	$RS(x_1 \cup X_3), RS(x_2 \cup X_3), RS(X_1),$ $RS(X_2), RS(X_1 \cup X_3), RS(X_2 \cup X_3)$	6
P_3	$RS(X_3)$	1
P_4	$\emptyset$	0
P_5	$RS(x_1 x_2)$	1
P_6	$\emptyset$	0
P_7	$RS(X_1 \cup X_2), RS(x_1 \cup X_2), RS(X_1 \cup x_2),$ $RS(x_1 \cup X_2 \cup X_3), RS(X_1 \cup x_2 \cup X_3),$ $RS(x_1 \cup x_2 \cup X_3)$	6

The partition graph for the above mentioned Example 3.2.1 is given below

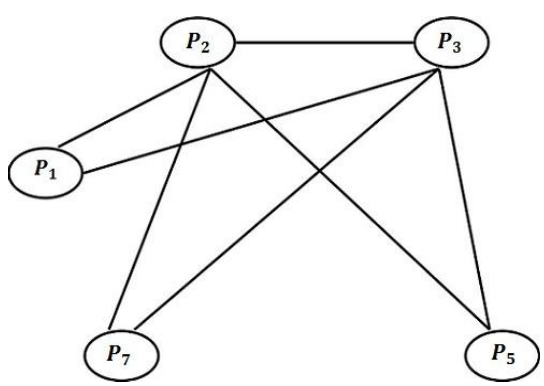

Figure 5: Partition Graph for $n = 3$ and $m = 2$

Example 3.2.2

From Illustration 2 in section 2, we examine the elements in each partition of $G(Z^*(J))$ using table 3.

Table 3: Partitions and Cardinalities of $G(Z^*(J))$for $n = m = 3$

Partitions (P_i)	**Elements**	**Cardinality**
P_1	$RS(x_1), RS(x_2), RS(x_3)$	3
P_2	$RS(X_1), RS(X_2), RS(X_3)$	3
P_3	$\emptyset$	0
P_4	$RS(x_1 \cup x_2), RS(x_1 \cup x_3), RS(x_2 \cup x_3)$	3
P_5	$RS(x_1 \cup x_2 \cup x_3)$	1
P_6	$RS(X_1 \cup X_2), RS(X_1 \cup X_3), RS(X_2 \cup X_3)$ $RS(x_1 \cup X_2), RS(x_1 \cup X_2), RS(x_1 \cup X_3)$ $RS(X_1 \cup x_3), RS(x_2 \cup X_3), RS(X_2 \cup x_3)$	9
P_7	$RS(x_1 \cup X_2 \cup X_3), RS(X_1 \cup x_2 \cup X_3), RS(X_1 \cup X_2 \cup x_3)$ $RS(x_1 \cup x_2 \cup X_3), RS(x_1 \cup X_2 \cup x_3), RS(X_1 \cup x_2 \cup x_3)$	6

The partition graph for t Example 3.2.2 is given below

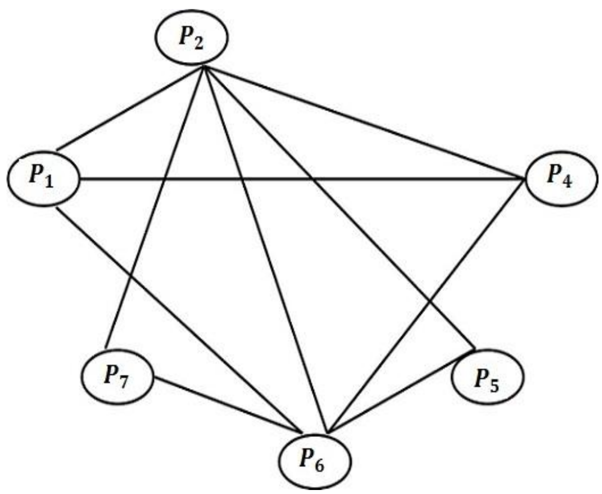

Figure 6: Partition Graph for $n = m = 3$

3.2 Wiener Index of $G(Z^*(J))$ using Partition Graph

In this section we calculate the wiener index of $G(Z^*(J))$ using Partition Graph $P(Z^*(J))$.

Definition 3.3.1:

Let $G(Z^*(J))$ be a Rough co-zero divisor graph. The wiener index $W(Z^*(J))$ of $G(Z^*(J))$ defined by $W(Z^*(J)) = \sum_{RS(x_i),RS(x_j)\in V(Z^*(J))} d\left(RS(x_i), RS(x_j)\right)$, where $d\left(RS(x_i), RS(x_j)\right)$ is the distance between the elements $RS(x_i)$ and $RS(x_j)$ in $G(Z^*(J))$.

To make the calculation simpler, if $d\left(RS(x_i), RS(x_j)\right)$ is taken for calculation then $d\left(RS(x_j), RS(x_i)\right)$ is not taken into account as both are one and the same.

Connectedness between the Partitions

In this section we compute the distances from one partition to every other partition. The main advantage of the partition graph is that all the vertices of one partition will behave similarly. Hence the distance between the vertices of $G\big(Z^*(J)\big)$ can be obtained by calculating distance between the partition in $P\big(Z^*(J)\big)$.Therefore we calculate the distance from $P_1$to P_2, P_3, P_4, P_5, P_6 and P_7 the distance from $P_2$to P_3, P_4, P_5, P_6 and P_7 ... and the distance from $P_6$to P_7. Following propositions will detail the distance from one partition to another.

Proposition 3.3.1:

The distance between the vertices of P_1 to the vertices of P_2 is given by

$$|P_1|\{1.(|P_2| - 2^{n+1-m} + 1) + 2.(-2^{n-m+1} + 1)\}$$
$$= m\{1.(2m(2^{n-m}) - m - 2^{n+1-m} + 1) + 2.(1-2^{n-m+1})\}$$

Where $P_1 = RS(x_i)$; $P_2 = RS(x_i \cup M') \cup RS(X_i \cup M)$

Proof:

For each $i = 1,2,..m$ $RS(x_i)$ is connected to $RS(x_i \cup M') \cup RS(X_i \cup M)$ for $i \neq j$ and hence the distance is one. Therefore we have $(1).|P_1|(|P_2| - (2^{n-m} - 1) - ((2^{n-m}))$

For $i = j$ the distance between $RS(x_i)$ to $RS(x_i \cup M') \cup RS(X_i \cup M)$ is 2. Thus we have $(2).|P_1|(-(2^{n-m} - 1) - (2^{n-m}))$.

Therefore the sum of such distance is given by $|P_1|\{1.(|P_2| - 2^{n+1-m} + 1) + 2.(-2^{n-m+1} + 1)\} = m\{1.(2m(2^{n-m}) - m - 2^{n+1-m} + 1) + 2.(1-2^{n-m+1})\}$

Proposition 3.3.2:

The distance between the vertices of P_1 to the vertices of P_3 is given by $1.(|P_1|.|P_3|) = 1.(m.(2^{n-m} - 1))$, where $P_1 = RS(x_i)$ and $P_3 = \{RS(Y)|Y \in M'\}$

Proof:

Since for $i = 1,2 \ldots m$ each element in the partition P_1 is connected to each element in P_3. Hence the distance is $1.(|P_1|.|P_3|) = 1.(m.(2^{n-m} - 1))$

Proposition 3.3.3:

The distance between the vertices of P_1 to the vertices of P_4 is given by

$|P_1|\{1.(|P_4|-(2^{m-1}-2))+2.((2^{m-1}-2)\}=m\{1.((2^m-(m+2))-(2^{m-1}-2))+2.(2^{m-1}-2)\}$ Where $P_1=RS(x_i)$ and $P_4=RS(x_1,x_2,\dots x_r)$

Proof:

Considering the elements in P_1 there exist $(2^m-(m+2)-2^{m-1}+2)$ elements are connected to P_4 with distance 1 and $2^{m-1}+2$ elements are connected with distance 2.

Thus the distance from P_1 to P_4 is $|P_1|\{1.(|P_4|-(2^{m-1}-2))+2.((2^{m-1}-2)\}=m\{1.((2^m-(m+2))-(2^{m-1}-2))+2.(2^{m-1}-2)\}$

Proposition 3.3.4:

The distance between the vertices of P_1 to the vertices of P_5 is given by $2.(|P_1|.|P_5|)=2.m$

Where $P_1=RS(x_i)$ and $P_4=RS(x_1,x_2,\dots x_m)$

Proof:

Being $i=1,2\dots m$ none of the elements in P_1 is connected to P_5 with distance 1.

Hence $2.(|P_1|.|P_5|)=2m$. Which means that every elements in P_1 is not connected to P_5.

Proposition 3.3.5:

The distance between the vertices of P_1 to the vertices of P_6 is given by

$$|P_1|\{1.\left[(m-1)C_r\left(2^{-(-n+m)}-1\right)+\left(2^{-(m-n)}\right)((m-1)C_r+\sum_{r=2}^{m-1}\left((m-1)C_1+(m-1)C_2+\dots+(m-1)C_{r-1}\right)\right])+2.\left[|P_6|-(m-1)C_r(2^{n-m}-1)+(m-1)C_r(2^{n-m})+(2^{n-m})\sum_{r=2}^{m-1}\left((m-1)C_1+(m-1)C_2+\dots+(m-1)C_{r-1}\right)\right]\}=m\{1.\left[(m-1)C_r(2^{n-m}-1)+(m-1)C_r(2^{n-m})+(2^{n-m})\sum_{r=2}^{m-1}\left((m-1)C_1+(m-1)C_2+\dots+(m-1)C_{r-1}\right)\right]+2.\left[2^m-(2+m)(2^{n+1-m}-1)+3(2^{n-m})(3^{m-1}-2^m+1)-(m-1)C_r(2^{n-m}-1)+(m-1)C_r(2^{n-m})+(2^{n-m})\sum_{r=2}^{m-1}\left((m-1)C_1+(m-1)C_2+\dots+(m-1)C_{r-1}\right)\right]\}$$

Where $P_1=RS(x_i)$ and $P_6=RS(x_1,x_2,\dots x_r\cup M')\cup RS(X_1,X_2,\dots X_r\cup M)\cup RS(Q_r\cup M)$

Proof:

In view of partition P_6 has $RS(x_1,x_2,\dots x_r\cup M')\cup RS(X_1,X_2,\dots X_r\cup M)\cup RS(Q_r\cup M)$ elements with cardinality $2^m-(m+2)(2^{n-m+1}-1)+3(2^{n-m})(3^{m-1}-2^m+1)$.

Since $RS(x_i)$ is connected to $RS(x_1,x_2,\dots x_r\cup M')$ accompanying $2^m-(m+2)(2^{n-m}-1)$ with distance 1, $RS(x_i)$ is connected to $RS(X_1,X_2,\dots X_r\cup M)$ accompanying $2^m-(m+2)(2^{n-m})$ with distance 1 and $RS(x_i)$ is connected to $RS(Q_r\cup M)$ accompanying

$(2^{n-m})(3^{m-1}-2^m+1)$ with distance 1. Remaining elements in P_6 are connected $RS(x_i)$ with distance 2.

On that account we have the distance from P_1 to P_6 is

$|P_1|\{1.[(m-1)C_r(2^{-(-n+m)}-1)+(2^{-(m-n)})((m-1)C_r+\sum_{r=2}^{m-1}((m-1)C_1+(m-1)C_2+\cdots+(m-1)C_{r-1})])+2.[|P_6|-(m-1)C_r(2^{n-m}-1)+(m-1)C_r(2^{n-m})+(2^{n-m})\sum_{r=2}^{m-1}((m-1)C_1+(m-1)C_2+\cdots+(m-1)C_{r-1})]\}=m\{1.[(m-1)C_r(2^{n-m}-1)+(m-1)C_r(2^{n-m})+(2^{n-m})\sum_{r=2}^{m-1}((m-1)C_1+(m-1)C_2+\cdots+(m-1)C_{r-1})]+2.[2^m-(2+m)(2^{n+1-m}-1)+3(2^{n-m})(3^{m-1}-2^m+1)-(m-1)C_r(2^{n-m}-1)+(m-1)C_r(2^{n-m})+(2^{n-m})\sum_{r=2}^{m-1}((m-1)C_1+(m-1)C_2+\cdots+(m-1)C_{r-1})]\}$

Proposition 3.3.6:

The distance between the vertices of P_1 to the vertices of P_7 is given by

$$2.(|P_1|.|P_7|)=2.(m.2^{n-m}(2^m)-2),$$

Where $P_1=RS(x_i)$ and $P_6=RS(x_1,x_2,\dots x_m\cup M')\cup RS(X_1,X_2,\dots X_m\cup M)\cup RS(Q_m\cup M)$

Proof:

Being $i=1,2\dots m$ none of the elements in P_1 is connected to P_7 with distance 1. Therefore it is connected through another element so the distance 2.

Hence $2.(|P_1|.|P_7|)=2.(m.2^{n-m}(2^m)-2)$.

Proposition 3.3.7:

The distance between the vertices of P_2 to the vertices of P_3 is given by $1.(|P_2|.|P_3|)=1.(2^{n-m}(2m+1)-(m+1))$

Where $P_2=RS(x_i\cup M')\cup RS(X_i\cup M)$; $P_3=\{RS(Y)|Y\in M'\}$

Proof:

In P_2 we posses m equivalence classes and P_3 have $n-m$ equivalence classes. By using the definition of Rough co-zero divisor graph every elements in P_2 is connected to every elements in P_3. Hence P_2 is adjacent to P_3 with distance 1.

Thus we have $1.(|P_2|.|P_3|)=1.(2^{n-m}(2m+1)-(m+1))$.

In a similar manner by the definition of partition graph, partition P_2 is connected to P_4, P_5, P_6 and P_7 accompanied by distance 1.

Proposition 3.3.8:

The distance between the vertices of P_2 to the vertices of P_4 is given by $1.(|P_2|.|P_4|) = 1.[(2m(2^{n-m}) - m)(2^m - (m+2))]$

Where$P_2 = RS(x_i \cup M') \cup RS(X_i \cup M)$; $P_4 = RS(x_1, x_2 \dots x_r)$

Proof:

Can be proved by direct verification.

Proposition 3.3.9:

The distance between the vertices of P_2 to the vertices of P_5 is given by $1.(|P_2|.|P_5|) = 1.(2m(2^{n-m}) - m))$ where$P_2 = RS(x_i \cup M') \cup RS(X_i \cup M)$; $P_5 = RS(x_1, x_2 \dots x_m)$

Proof:

Can be proved by direct verification.

Proposition 3.3.10:

The distance between the vertices of P_2 to the vertices of P_6 is given by $1.(|P_2|.|P_6|) = 1.[(2m(2^{n-m}) - m)).(2^m - (m+2)(2^{n-m+1} - 1) + 3(2^{n-m})(3^{m-1} - 2^m + 1))]$

Where $P_2 = RS(x_i \cup M') \cup RS(X_i \cup M)$; $P_6 = RS(x_1, x_2 \dots x_r \cup M') \cup RS(X_1, X_2 \dots X_r \cup M) \cup RS(Q_r \cup M)$

Proof:

Can be proved by direct verification.

Proposition 3.3.11:

The distance between the vertices of P_2 to the vertices of P_7 is given by $1.(|P_2|.|P_7|) = 1.[(2m(2^{n-m}) - m)).(2^{n-m}(2^m) - 2)]$

Where $P_2 = RS(x_i \cup M') \cup RS(X_i \cup M)$; $P_7 = RS(x_1, x_2 \dots x_m \cup M') \cup RS(X_1, X_2 \dots X_m \cup M) \cup RS(Q_m \cup M)$

Proof:

Can be proved by direct verification.

Proposition 3.4.12:

The distance between the vertices of P_3 to the vertices of P_4 is given by $1.(|P_3|.|P_4|) = 1.[(2^{n-m}-1).(2^m-(m+2))]$

Where $P_3 = \{RS(Y)|Y \in M'\}; P_4 = RS(x_1, x_2 \dots x_r)$

Proof:

Since the elements in P_3 contains maximum degree, which indicates that the distance from P_3 to P_4, P_5, P_6 and P_7 is 1. Hence the distance from P_3 to P_4 is $1.(|P_3|.|P_4|) = 1.[(2^{n-m}-1).(2^m-(m+2))]$

Proposition 3.3.13:

The distance between the vertices of P_3 to the vertices of P_5 is given by $1.(|P_3|.|P_5|) = 1.(2^{n-m}-1)$ where $P_3 = \{RS(Y)|Y \in M'\}; P_5 = RS(x_1, x_2 \dots x_m)$

Proof:

The proof is obvious from the Proposition 3.3.12

Proposition 3.3.14:

The distance between the vertices of P_3 to the vertices of P_6 is given by $1.(|P_3|.|P_6|) = \left[(2^m-(m+2)(2^{n+1-m}-1)(2^{n-m}-1)+3\left(2^{-(-n+m)}\right)(3^{m-1}+1-2^m))\right]$ where

$P_3 = \{RS(Y)|Y \in M'\}; P_6 = RS(x_1, x_2 \dots x_r \cup M') \cup RS(X_1, X_2 \dots X_r \cup M) \cup RS(Q_r \cup M)$

Proof:

Can be proved by direct verification.

Proposition 3.3.15:

The distance between the vertices of P_3 to the vertices of P_7 is given by $1.(|P_3|.|P_7|) = 1.[(2^{n-m}-1).(2^{n-m}(2^m)-2)]$ where $P_3 = \{RS(Y)|Y \in M'\}$; $P_7 = RS(x_1, x_2 \dots x_m \cup M') \cup RS(X_1, X_2 \dots X_m \cup M) \cup RS(Q_m \cup M)$

Proof:

Can be proved by direct verification.

Proposition 3.3.16:

The distance between the vertices of P_4 to the vertices of P_5 is given by $2.(|P_4|.|P_5|) = 1.[(2^m-m-2)]$ where $P_4 = RS(x_1, x_2 \dots x_r)$ and $P_5 = RS(x_1, x_2 \dots x_m)$

Proof:

None of the elements in P_4 is connected to P_5 with distance 1 because $1 < r < m$.

Hence the distance from P_4 to P_5 is $2.(|P_4|.|P_5|) = 1.[(2^m - m - 2)]$.

Proposition 3.3.17:

The distance between the elements of P_4 to the vertices of P_6 is given by $|P_4|\left[1.\left(|P_6| - (* + ** + ***)\right) + 2.(* + ** + ***)\right] = 2^m - (m+2)\left[1.\left(2^m - (m+2)(2^{n+1-m+1} - 1 + 3(2^{n-m})(3^{m-3-2} - 2^m + 1) - (* + ** + ***)\right) + 2.(* + ** + ***)\right]$

where $P_4 = RS(x_1, x_2 \dots x_r)$ and $P_6 = RS(x_1, x_2 \dots x_r \cup M') \cup RS(X_1, X_2 \dots X_r \cup M) \cup RS(Q_r \cup M)$

Proof:

Considering the partition P_4 is of the form $RS(x_1, x_2 \dots x_r)$ with $2^m - (m+2)$ elements, where $1 < r < m$ and partition P_6 is an association of three set namely the first set is $RS(x_1, x_2 \dots x_r \cup M')$, the second set is $RS(X_1, X_2 \dots X_r \cup M)$ and the third set is $RS(Q_r \cup M)$.

For $r = 2,3 \dots (m-1)$ the total number of two element category in P_4 is $\frac{m(m-1)}{2}$, the total number of three element category in P_4 is $\frac{m(m-1)(m-2)}{2} - (1.2 + 2.3 + \dots + (m-2)(m-1))$ and so on beyond any doubt $(m-1)$ number of elements in 1.

Case1:

Two element category in P_4 is connected to the first set of P_6 with distance 2 and is given by $\left[\frac{m(m-1)}{2}\right].[(m-2)C_0 + (m-2)C_1 + \dots + (m-2)C_{m-3}](2^{n-m} - 1)$

Correspondingly the three element category in P_4 is connected to the first set of P_6 with distance 2 and is given by

$\left[\frac{m(m-1)(m-2)}{2} - 1.2 + 2.3 + \cdots (m-2)(m-1)\right].[(m-3)C_0 + (m-3)C_1 + \dots + (m-3)C_{m-4}](2^{n-m} - 1)$ and so on eventually $(m-1)$ element category in P_4 is connected to the first set of P_6 with distance 2 and is given by $\left[m\left(m(m-1)\right).(2^{n-m} - 1)\right]$

Adding all we get

$$\left\{\left[\frac{m(m-1)}{2}\right].[(m-2)C_0 + (m-2)C_1 + \dots + (m-2)C_{m-3}] + \left[\frac{m(m-1)(m-2)}{2} - 1.2 + 2.3 + \cdots (m-2)(m-1)\right].[(m-3)C_0 + (m-3)C_1 + \dots + (m-3)C_{m-4}] + \dots + 1.\left[m\left(m(m-1)\right).(2^{n-m} - 1)\right]\right\} \dots \dots \dots \dots \dots (*)$$

Case 2:

Two element category in P_4 is connected to the first set of P_6 with distance 2 and is given by $\left[\frac{m(m-1)}{2}\right].[(m-2)C_0+(m-2)C_1+\cdots+(m-2)C_{m-3}](2^{n-m})$

Correspondingly the three element category in P_4 is connected to the first set of P_6 with distance 2 and is given by

$\left[\frac{m(m-1)(m-2)}{2}-1.2+2.3+\cdots(m-1)(m-2)\right].[(m-3)C_0+(m-3)C_1+\cdots+(m-3)C_{m-4}](2^{n-m})$ and so on eventually $(m-1)$ element category in P_4 is connected to the first set of P_6 with distance 2 and is given by $\left[m\big(m(m-1)\big).(2^{n-m})\right]$

Adding all we get

$$\left\{\left[\frac{m(m-1)}{2}\right].[(m-2)C_0+(m-2)C_1+\cdots+(m-2)C_{m-3}]+\left[\frac{m(m-1)(m-2)}{2}-1.2+2.3+\cdots(m-1)(m-2)\right].[(m-3)C_0+(m-3)C_1+\cdots+(m-3)C_{m-4}]+\cdots+\left[m\big(m(m-1)\big).(2^{n-m})\right]\right\}\ldots\ldots\ldots\ldots\ldots\ldots\ldots(**)$$

Case 3:

Two element category in P_4 is connected to the first set of P_6 with distance 2 and is given by $\left[\frac{m(m-1)}{2}\right]\left[\sum_{k=2}^{m-1}\sum_{i=1}^{k-1}kC_i\right](2^{n-m})$

Correspondingly the three element category in P_4 is connected to the first set of P_6 with distance 2 and is given by

$\left[\frac{m(m-1)(m-2)}{2}-1.2+2.3+\cdots(m-2)(m-1)\right].\left[\sum_{k=3}^{m-1}\sum_{i=1}^{k-1}kC_i\right](2^{n-m})$ and so on eventually $(m-1)$ element category in P_4 is connected to the first set of P_6 with distance 2 and is given by $\left[m\big(m(m-1)\big).(2^{n-m})\right]$

Adding all we get

$$\left\{\left[\frac{m(m-1)}{2}\right]\left[\sum_{k=2}^{m-1}\sum_{i=1}^{k-1}kC_i\right]+\left[\frac{m(m-1)(m-2)}{2}-1.2+2.3+\cdots(m-2)(m-1)\right].\left[\sum_{k=3}^{m-1}\sum_{i=1}^{k-1}kC_i\right]+\cdots+1\right\}(2^{n-m})\ldots\ldots\ldots\ldots(***)$$

Therefore the distance between the elements of P_4 to the vertices of P_6 is given by $|P_4|\left[1.\big(|P_6|-(*+**+***)\big)+2.(*+**+***)\right]$.

Proposition 3.3.18:

The distance between the vertices of P_4 to the vertices of P_7 is given by $2.(|P_4|.|P_7|) = 2.[(2^m - (m+2)).(2^{n-m}(2^m) - 2)]$ where $P_4 = RS(x_1, x_2 \dots x_r)$

and $P_7 = RS(x_1, x_2 \dots x_m \cup M') \cup RS(X_1, X_2 \dots X_m \cup M) \cup RS(Q_m \cup M)$

Proof:

None of the elements in P_4 is connected to P_7 with distance 1.

Hence $2.(|P_4|.|P_7|) = 2.[(2^m - (m+2)).(2^{n-m}(2^m) - 2)]$.

Proposition 3.3.19:

The distance between the vertices of P_5 to the vertices of P_6 is given by $1.(|P_5|.|P_6|) = 1.[(2^m - (m+2)(2^{n+1-m} - 1) + 3(2^{n-m})(3^{m-5+4} - 2^m + 1)]$where $P_5 = RS(x_1, x_2 \dots x_m)$

and $P_7 = RS(x_1, x_2 \dots x_r \cup M') \cup RS(X_1, X_2 \dots X_r \cup M) \cup RS(Q_r \cup M)$

Proof:

In consequence P_5 has single element and it is connected to all the elements in P_6. Thus the distance from P_5 to P_6 is

$1.(|P_5|.|P_6|) = 1.[(2^m - (m+2)(2^{n+1-m} - 1) + 3(2^{n-m})(3^{m-5+4} - 2^m + 1)]$.

Proposition 3.3.20:

The distance between the vertices of P_5 to the vertices of P_7 is given by $2.(|P_5|.|P_6|) = 1.[2^{n-m}(2^m) - 2]$ where $P_5 = RS(x_1, x_2 \dots x_m)$

and $P_7 = RS(x_1, x_2 \dots x_m \cup M') \cup RS(X_1, X_2 \dots X_m \cup M) \cup RS(Q_m \cup M)$

Proof:

Element in P_5 is connected to every elements in P_7 with distance 2. Hence $2.(|P_5|.|P_7|) = 2.[2^{n-m}(2^m) - 2]$.

Proposition 3.3.21:

The distance between the elements of P_6 to the elements of P_7 is given by $1.(|P_6|.|P_7|) = 2.[(2^m - (m+2)(2^{n-m+1} - 1) + 3(2^{n-m})(3^{m-1} - 2^m + 1).(2^{n-m}(2^m) - 2)]$.

Where $P_6 = RS(x_1, x_2 \dots x_r \cup M') \cup RS(X_1, X_2 \dots X_r \cup M) \cup RS(Q_r \cup M)$

$P_7 = RS(x_1, x_2 \dots x_m \cup M') \cup RS(X_1, X_2 \dots X_m \cup M) \cup RS(Q_m \cup M)$

Proof:

All the elements in P_6 is connected with P_7 with distance $1.(|P_6|.|P_7|) = 1.[(2^m - (m+2)(2^{n-m+1} - 1) + 3(2^{n-m})(3^{m-1} - 2^m + 1).(2^{n-m}(2^m) - 2)]$

Theorem 3.3.1:

The wiener index $W(Z^*(J))$ of Rough co-zero divisor graph $G(Z^*(J))$, when $n \neq m$ is$(A) + (B) + (C)$ where

$A = |P_1|\{1.\big[|P_2| + |P_3| + |P_4| - (2^n - 3) + (m-1)C_r\big(2^{-(-n+m)} - 1\big) + (2^{n-m})((m-1)C_r + \sum_{r=2}^{m-1}\big((m-1)C_1 + (m-1)C_2 + \cdots + (m-1)C_{r-1}\big))\big] + 2.\big[2^{m-1} - 2^{n-m+2} + 1 + |P_5| + |P_6| - (m-1)C_r\big(2^{-(-n+m)} - 1\big) + (2^{n-m})((m-1)C_r + \sum_{r=2}^{m-1}\big((m-1)C_1 + (m-1)C_2 + \cdots + (m-1)C_{r-1}\big))\big(2^{-(-n+m}\big) + |P_7|\big]\} + |P_2|\{1.[|P_3| + |P_4| + |P_5| + |P_6| + |P_7|]\} + |P_3|\{1.[|P_4| + |P_5| + |P_6| + |P_7|]\} + |P_4|\{1.\big[\big(|P_6| - (* + ** + ***)\big)\big] + 2.[|P_5| + (* + ** + ***)].+|P_7|\big\} + |P_5|\{1.|P_6| + 2.|P_7|\} + |P_6|\{1.|P_7|\}$

$B = \frac{1}{2}\{|P_1|\,(|P_1| - 1) + |P_2|\,(|P_2| - 1) + |P_3|\,(|P_3| - 1) + |P_5|\,(|P_5| - 1) + |P_6|\,(|P_6| - 1) + |P_7|\,(|P_7| - 1)\}$

$C = 1.\left[\frac{(m-1)m}{2}\right] + \left[\frac{m(m-1)(m-2)}{2} - 1.2 + 2.3 + \cdots + (m-1)(m-2)\right] + \cdots.+1 + 2.\Big\{\big(2^m - (m+2)\big) - \left[\frac{(m-1)m}{2}\right] + \left[\frac{(m-1)(m-2)m}{2} - 1.2 + 2.3 + \cdots + (m-1)(m-2)\right] + \cdots.+1\Big\}$

Proof:

The wiener index of $G(Z^*(J))$ is the sum of the distance from every vertex to every other vertex and hence from Proposition 3.3.1 to Proposition 3.3.21 we have

$|P_1|\{1.\big[|P_2| + |P_3| + |P_4| - (2^n - 3) + (m-1)C_r\big(2^{-(-n+m} - 1\big) + (m-1)C_r + \sum_{r=2}^{m-1}\big((m-1)C_1 + (m-1)C_2 + \cdots + (m-1)C_{r-1}\big)(2^{n-m})\big] + 2.\big[2^{m-1} - 2^{n-m+2} + 1 + |P_5| + |P_6| - (m-1)C_r\big(2^{-(-n+m)} - 1\big) + ((m-1)C_r + \sum_{r=2}^{m-1}\big((m-1)C_1 + (m-1)C_2 + \cdots + (m-1)C_{r-1}\big))(2^{n-m}) + |P_7|\big]\} + |P_2|\{1.[|P_3| + |P_4| + |P_5| + |P_6| + |P_7|]\} + |P_3|\{1.[|P_4| + |P_5| + |P_6| + |P_7|]\} + |P_4|\{1.\big[\big(|P_6| - (* + ** + ***)\big)\big] + 2.[|P_5| + (* + ** + ***)].+|P_7|\big\} + |P_5|\{1.|P_6| + 2.|P_7|\} + |P_6|\{1.|P_7|\} \ldots\ldots\ldots\ldots.(A)$

Proposition 3.3.1 to 3.3.21, gives the distance from one partition to every other partition. Hence equation (A) gives the distance from the vertices of one partition to vertices of every other partition. Also note that the vertices of P_1, P_2, P_3, P_5, P_6 and P_7 form a complete subgraph of $G(Z^*(J))$. Therefore the distances between the vertices in each of these partition in each of these partition are one. Hence the sum of such distance is given by $\frac{n(n-1)}{2}$ where $n = |P_i|$ for $i = 1,2,3,5,6,7$ which is calculated by $\frac{|P_i|\,(|P_i| - 1)}{2}$

(i.e) $\frac{|P_1|(|P_1|-1)}{2}+\frac{|P_2|(|P_2|-1)}{2}+\frac{|P_3|(|P_3|-1)}{2}+\frac{|P_5|(|P_5|-1)}{2}+\frac{|P_6|(|P_6|-1)}{2}+\frac{|P_7|(|P_7|-1)}{2}$

$\frac{1}{2}\{|P_1|(|P_1|-1)+|P_2|(|P_2|-1)+|P_3|(|P_3|-1)+|P_5|(|P_5|-1)+|P_6|(|P_6|-1)+|P_7|(|P_7|-1)\} \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots..(B)$

Now when $i=4$, we need to find the distance between the vertices in $|P_4|$. Note that $P_4 = RS(x_1, x_2 \dots x_r)$. Hence we calculate the distance within the vertices of P_4 is

$1.\left[\frac{(m-1)m}{2}\right]+\left[\frac{(m-1)(m-2)m}{2}-1.2+2.3+\cdots+(m-1)(m-2)\right]+\cdots.+1+2.\left\{\left(2^m-(m+2)\right)-\left[\frac{(m-1)m}{2}\right]+\left[\frac{m(m-1)(m-2)}{2}-1.2+2.3+\cdots+(m-1)(m-2)\right]+\cdots.+1\right\}\dots\dots(C)$

The wiener index of $G(Z^*(J))$ is obtained as by using $(A)+(B)+(C)$ we get

$W\left(Z^*(J)\right)=|P_1|\{1.\left[|P_2|+|P_3|+|P_4|-(2^n-3)+(m-1)C_r(2^{n-m}-1)+(m-1)C_r(2^{n-m})+(2^{n-m})\sum_{r=2}^{m-1}\left((m-1)C_1+(m-1)C_2+\cdots+(m-1)C_{r-1}\right)\right]+2.\left[2^{m-1}-2^{n-m+2}+1+|P_5|+|P_6|-(m-1)C_r(2^{n-m}-1)+(m-1)C_r(2^{n-m})+(2^{n-m})\sum_{r=2}^{m-1}\left((m-1)C_1+(m-1)C_2+\cdots+(m-1)C_{r-1}\right)+|P_7|\right]\}+|P_2|\{1.[|P_3|+|P_4|+|P_5|+|P_6|+|P_7|]\}+|P_3|\{1.[|P_4|+|P_5|+|P_6|+|P_7|]\}+|P_4|\{1.\left[\left(|P_6|-(*+**+***)\right)\right]+2.[|P_5|+(*+**+***)].+|P_7|\}+|P_5|\{1.|P_6|+2.|P_7|\}+|P_6|\{1.|P_7|\}+\frac{1}{2}\{|P_1|(|P_1|-1)+|P_2|(|P_2|-1)+|P_3|(|P_3|-1)+|P_5|(|P_5|-1)+|P_6|(|P_6|-1)+|P_7|(|P_7|-1)\}+1.\left[\frac{m(m-1)}{2}\right]+\left[\frac{m(m-1)(m-2)}{2}-1.2+2.3+\cdots(m-2)(m-1)\right]+\cdots.+1+2.\left\{\left(2^m-(m+2)\right)-\left[\frac{m(m-1)}{2}\right]+\left[\frac{m(m-1)(m-2)}{2}-1.2+2.3+\cdots(m-2)(m-1)\right]+\cdots.+1\right\}\dots..(I)$

Theorem 3.3.2:

The wiener index $W(Z^*(J))$ of Rough co-zero divisor graph $G(Z^*(J))$, when $n=m$ is

$|P_1|\{1.\left[|P_2|+|P_4|-(2^n-3)+(m-1)C_r(2^{n-m}-1)+(m-1)C_r(2^{n-m})+(2^{n-m})\sum_{r=2}^{m-1}\left((m-1)C_1+(m-1)C_2+\cdots+(m-1)C_{r-1}\right)\right]+2.\left[2^{m-1}-2^{n-m+2}+1+|P_5|+|P_6|-(m-1)C_r(2^{n-m}-1)+(m-1)C_r(2^{n-m})+(2^{n-m})\sum_{r=2}^{m-1}\left((m-1)C_1+(m-1)C_2+\cdots+(m-1)C_{r-1}\right)+|P_7|\right]\}+|P_2|\{1.[+|P_4|+|P_5|+|P_6|+|P_7|]\}+|P_4|\{1.\left[\left(|P_6|-(*+**+***)\right)\right]+2.[|P_5|+(*+**+***)].+|P_7|\}+|P_5|\{1.|P_6|+2.|P_7|\}+|P_6|\{1.|P_7|\}+\frac{1}{2}\{|P_1|(|P_1|-1)+|P_2|(|P_2|-1)++|P_5|(|P_5|-1)+|P_6|(|P_6|-1)+|P_7|(|P_7|-1)\}+1.\left[\frac{m(m-1)}{2}\right]+\left[\frac{m(m-1)(m-2)}{2}-1.2+2.3+\cdots(m-2)(m-1)\right]+\cdots.+1+2.\left\{\left(2^m-(m+2)\right)-\left[\frac{m(m-1)}{2}\right]+\left[\frac{m(m-1)(m-2)}{2}-1.2+2.3+\cdots(m-2)(m-1)\right]+\cdots.+1\right\}\dots\dots\dots\dots.(II)$

Proof:

Now we calculate the wiener index of Rough co-zero divisor graph $G(Z^*(J))$ when $n = m$. Since $P_3 = \emptyset$. Therefore by a similar calculation on leaving P_3 the wiener index of $G(Z^*(J))$ is

$$W\big(Z^*(J)\big) = |P_1|\big\{1.\big[|P_2| + |P_4| - (2^n - 3) + (m-1)C_r(2^{n-m} - 1) + (m-1)C_r(2^{n-m}) + (2^{n-m})\textstyle\sum_{r=2}^{m-1}\big((m-1)C_1 + (m-1)C_2 + \cdots + (m-1)C_{r-1}\big)\big] + 2.\big[2^{m-1} - 2^{n-m+2} + 1 + |P_5| + |P_6| - (m-1)C_r(2^{n-m} - 1) + (m-1)C_r(2^{n-m}) + (2^{n-m})\textstyle\sum_{r=2}^{m-1}\big((m-1)C_1 + (m-1)C_2 + \cdots + (m-1)C_{r-1}\big) + |P_7|\big]\big\} + |P_2|\{1.[+|P_4| + |P_5| + |P_6| + |P_7|]\} + |P_4|\big\{1.\big[\big(|P_6| - (* + ** + ***)\big)\big] + 2.[|P_5| + (* + ** + ***)].+|P_7|\big\} + |P_5|\{1.|P_6| + 2.|P_7|\} + |P_6|\{1.|P_7|\} + \frac{1}{2}\{|P_1|\,(|P_1| - 1) + |P_2|\,(|P_2| - 1) + +|P_5|\,(|P_5| - 1) + |P_6|\,(|P_6| - 1) + |P_7|\,(|P_7| - 1)\} + 1.\left[\frac{m(m-1)}{2}\right] + \left[\frac{m(m-1)(m-2)}{2} - 1.2 + 2.3 + \cdots (m-2)(m-1)\right] + \cdots.+1 + 2.\left\{\big(2^m - (m+2)\big) - \left[\frac{m(m-1)}{2}\right] + \left[\frac{m(m-1)(m-2)}{2} - 1.2 + 2.3 + \cdots (m-2)(m-1)\right] + \cdots.+1\right\} \ldots\ldots\ldots\ldots.(II)$$

Example 3.3.1

From Example 3.2.1 we have $n = m = 3$ by substituting all the values in equation (II) we get

$$W\big(Z^*(J)\big) = 2\{1.(3) + 2.(3)\} + 1.(1.2) + 2(2.1) + 2.(2.6) + 1.(6.1) + 1.(6.1) + 1.(1.1) + 1.(1.6) + 2.(1.6) + (1 + 15 + 15)$$

$$= 18 + 2 + 4 + 24 + 6 + 6 + 36 + 1 + 6 + 12 + 31 = 146$$

when $n = m = 3$, the wiener index $W\big(Z^*(J)\big)$ of $G\big(Z^*(J)\big)$ is obtained by 146

Example 3.3.2

From Example 3.2.2 we have $n = m = 3$ by substituting all the values in equation (II) we get

$$W\big(Z^*(J)\big) = 3\{1.(3 - 2 + 1) + 2.(-2 + 1)\} + 3\{1.(3 - 2) + 2.(2)\} + 2(3.1) + 3\big\{1.(3) + 2.\big(9 - (3)\big)\big\} + 2.\{(3.6) + 1.\{(3.3) + 1.\{(3.1) + 1.\{(3.9) + 1.\{(3.6) + 2.\{(3.1) + \{1.18 + 2.9 + 2.\{(3.6) + 2.\{(1.6) + 1.\{(9.6)\} + \{1.3 + 1.3 + 1.3 + 1.0 + 1.36 + 1.15\}$$

$$= 12 + 15 + 6 + 45 + 36 + 9 + 3 + 27 + 18 + 6 + 36 + 36 + 9 + 12 + 54 + 60 = 384$$

when $n = m = 3$, the wiener index $W\big(Z^*(J)\big)$ of $G\big(Z^*(J)\big)$ is obtained by 384.

4.APPLICATION

Let U be the set of symptoms namely Dry cough (u_1), fever (u_2), shortness of breath (u_3), fatigue (u_4), runny nose (u_5), nasal congestion (u_6), diarrhea (u_7), body aches (u_8), sore throat (u_9), head ache (u_{10}), loss of appetite (u_{11}), new loss of taste or smell (u_{12}), respiratory issues (u_{13}), nausea (u_{14}), vomiting (u_{15}), high temperature (u_{16}), sleeping disorders (u_{17}), difficulty of breathing (u_{18}) and sudden weight loss (u_{19}).

$$X_1 = \{u_1, u_2\}; X_2 = \{u_3, u_{11}, u_{18}\}; X_3 = \{u_7, u_{14}, u_{15}\}; X_4 = \{u_5, u_{12}, u_{16}, u_{17}\};$$

$$X_5 = \{u_6, u_8, u_9, u_{13}\}; X_6 = \{u_4, u_{10}\}; X_7 = \{u_{19}\}$$

Let D be the set of diseases Influenza d_1, Common cold d_2, COVID-19 d_3, Pneumonia d_4, Asthma d_5, Acute bronchitis d_6, and Malaria d_7.

If $X = \{Cough, sleeping\ disorders\}$ then $RS(X) = (\emptyset, X_1 \cup X_1)$ means two symptoms together cannot confirm d_1 and d_3 but these two are the symptoms possibility of either d_1 or d_3. $X = \{Cough, cold, sore\ throat\}$ then $RS(X) = (X_1,\ X_1)$ means confirms d_3 . A similar interpretation can be given for all the Rough sets in T^*. If $RS(X)$ is any vertex in $G(Z^*(J))$ then among the symptoms in U we are able to identify a subset Y that does not have any common symptoms with X along with J such elements are connected in $G(Z^*(J))$. Also people who have a symptom associated with other symptom of disease is distance 2 and people who have a symptom not associated with other symptom of disease is distance 1.

5.CONCLUSION

The focus of this study was to develop an innovative methodology for finding Degree, Distance and Wiener index of a Rough Co-zero divisor graph using partition graph. All aforesaid concepts are illustrated through examples. Our forthcoming work is to explore this partition graph to Rough Co-zero divisor graph.

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REFERENCES

[1]. Ansari-Toroghy, H., Farshadifar, F., & Mahboobi-Abkenar, F. (2016). An ideal-based cozero-divisor graph of a commutative ring, pp. 45-54

[2]. Donno, A. (2017). Spectrum, distance spectrum, and Wiener index of wreath products of complete graphs. *Ars Mathematica Contemporanea*, 13(1), 207-225.

[3]. Dankelmann, P., Gutman, I., Mukwembi, S., & Swart, H. C. (2009). The edge-Wiener index of a graph. *Discrete Mathematics*, 309(10), 3452-3457.

[4]. Dankelmann, P., Morgan, M. J., Mukwembi, S., & Swart, H. C. (2014). On the eccentric connectivity index and Wiener index of a graph. *Quaestiones Mathematicae*, 37(1), 39-47.

[5]. Eliasi, M., Raeisi, G., & Taeri, B. (2012). Wiener index of some graph operations. *Discrete Applied Mathematics*, 160(9), 1333-1344.

[6]. Hua, H. (2020). Comparing multiplicative Wiener index with other graph invariants. *MATCH Commun. Math. Comput. Chem*, 83, 95-107.

[7]. Praba, B. (2021). Clique number and Girth of the Rough Co-zero Divisor Graph. *Turkish Journal of Computer and Mathematics Education (TURCOMAT)*, 12(11), 5459-5466.

[8]. Afkhami, M., & Khashyarmanesh, K. (2011). The Cozero-divisor Graph of a Commutative Ring. *Southeast Asian Bulletin of Mathematics*, 35(5), 935-944.

[9]. Praba, B., Manimaran, A., & Chandrasekaran, V. M. (2015). The zero divisor graph of a rough semiring. *International Journal of Pure and Applied Mathematics*, 98(5), 33-37.

[10]. Praba, B. (2021). Clique number and Girth of the Rough Co-zero Divisor Graph. *Turkish Journal of Computer and Mathematics Education (TURCOMAT)*, 12(11), 5459-5466.

[11]. Praba, B., & Obilia, X. A. B. (2019). An application of category graph in finding the Wiener index of rough ideal based rough edge Cayley graph. *Applied Mathematics and Information Sciences*, 313-323.

[12]. Pawlak, Z. (1982). Rough sets. *International journal of computer & information sciences*, 11(5), 341-356.

[13]. Xu, K., Liu, M., Das, K. C., Gutman, I., & Furtula, B. (2014). A survey on graphs extremal with respect to distance-based topological indices. *MATCH Commun. Math. Comput. Chem*, 71(3), 461-508.