Neural, Parallel, and Scientific Computations 30 (2022) No.1, 17-22

DETERMINING AN OPTIMAL SCHEDULING OF FREIGHT TRANSPORTATION TO MAXIMIZE REVENUE

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ABSTRACT. In this paper we present a nonlinear programming approach for selecting an optimal scheduling of freight transportation from sources to destinations to maximize revenue. The decision variables in the model are number of batches to be transported from sources to destinations. These decision variables take on integer values. In our model we treat them as continuous variables and add a nonlinear constraint which will enforce the variables to take on integer values at the optimal solution. The nonlinear constrained problem is transformed to an unconstrained optimization problem using a penalty method. We use gradient descent to find the optimal solution of the unconstrained optimization problem. A numerical example with two sources and two destinations is provided.

Key Words and Phrases. Penalty method, constrained and unconstrained optimization

1. Introduction

Freight delivery problem gained even more importance with recent supply chain disruptions, increase in cost of transportation, global warning awareness, and sustainability objectives. Cooperative freight logistic is one of the alternative framework that intends to minimize costs and drawbacks for all. The problem can be solved in various time intervals and sizes using game theoretic (combinatorial games). Formulations can be equivalently explained as programming and usually solved with mixed linear integer programming.

In the literature, there are number of game theoretic approaches ([11], [12]).

In this paper, we present a nonlinear programming approach to determine an optimal scheduling of freight transportation from sources to destinations. Items to be transported are grouped in batches. The decision variables in the nonlinear programming model presented are the number of batches from sources to destinations. These decision variables take on integer values. In our approach, we treat these decision variables as continuous and add a nonlinear constraint to our model to enforce them to take integer values within a specified tolerance limit at the optimal solution. The constrained optimization problem is transformed to an unconstrained minimization problem using a penalty method. A gradient decent method is used to find an optimal solution of the unconstrained optimization problem.

2. Problem Statement and Model

In this section we define the problem that we want to address in this paper and also provide the model that describes the problem.

Suppose we have s sources from where items in batches are to be picked up and delivered to d destinations.

Let $\beta(k), k = 1, \cdots, d$ be the number of batches to be delivered to destination k from all the s sources.

Received March 20, 2022
www.dynamicpublishers.org; www.dynamicpublishers.com;
https://doi.org/10.46719/npsc202230.01.02.

Suppose the maximum number of batches source i can transport is $\alpha(i), i = 1, \cdots, s$.
We assume that $\alpha(i) \geq \beta(k)$, $\forall i = 1, \cdots, s$, $\forall k = 1, \cdots, d$.
Let

1. $c(\ell, k) =$ cost of transporting a batch from source ℓ to destination k, $\ell = 1, \cdots, s$, $k = 1, \cdots, d$
2. $b(\ell, k) =$ cost of transportation (which include fuel, storage cost, and other transportation costs) of a batch from source ℓ to destination k, $\ell = 1, \cdots, s$, $k = 1, \cdots, d$
3. $x(\ell, k) =$ number of batches to be transported from source ℓ to destination k, $\ell = 1, \cdots, s$, $k = 1, \cdots, d$

The cost is to the suppliers at the sources who want to deliver batches of items to various destinations. For the freight transport companies this cost will be a revenue. From the freight transportation company's point of view, the problem is to maximize revenue following the definitions of $\beta(k)$ and $\alpha(i)$ described above. Our goal thus is to maximize

$$\max\{\sum_{\ell=1}^{s}\sum_{k=1}^{d} c(\ell, k)x(\ell, k) - \sum_{\ell=1}^{s}\sum_{k=1}^{d} b(\ell, k)x(\ell, k)\}$$

subject to the restrictions described above. The optimization model described below describes the problem.

$$\begin{aligned}
&\max\{\sum_{\ell=1}^{s}\sum_{k=1}^{d} c(\ell, k)x(\ell, k) - \sum_{\ell=1}^{s}\sum_{k=1}^{d} b(\ell, k)x(\ell, k)\}\\
&\text{subject to}\\
&1 - \epsilon \leq \sum_{i=1}^{\beta(k)} e^{-\Delta(x(\ell,k)-i)^2} \leq 1 + \epsilon, \quad k = 1, \cdots, d; \ell = 1, \cdots, s\\
&\sum_{\ell=1^s} x(\ell, k) \leq \beta(k), \quad k = 1, \cdots, d\\
&0 \leq \beta(k) \leq m(k), \quad k = 1, \cdots, d\\
&0 \leq x(\ell, k) \leq \alpha(\ell), \quad \ell = 1, \cdots, s; k = 1, \cdots, d
\end{aligned}$$

in the model above, The parameter Δ is a positive number to be chosen (eg 5, 10, 15 etc). The parameter ϵ is a very small positive number (eg 0.0005) to be chosen. For each k, $m(k)$ is an upper bound for $\beta(k)$ to be chosen. The first constraint is to enforce the decision variables $x(\ell, k)$ to take on integer values. Unless you match one of the i's only the best candidate will be close to 1.
We use a penalty method to convert the above constrained optimization problem to an unconstrained minimization problem as

follows.

$$\min -\sum_{\ell=1}^{s}\sum_{k=1}^{d} c(\ell,k)x(\ell,k) + \sum_{\ell=1}^{s}\sum_{k=1}^{d} b(\ell,k)x(\ell,k)$$

$$+L_1\sum_{\ell=1}^{s}\sum_{k=1}^{d}\omega\left(\sum_{i=1}^{\beta(k)} e^{-\Delta(x(\ell,k)-i)^2} - 1 - \epsilon\right)$$

$$+L_2\sum_{\ell=1}^{s}\sum_{k=1}^{d}\omega\left(1 - \epsilon - \sum_{i=1}^{\beta(k)} e^{-\Delta(x(\ell,k)-i)^2}\right)$$

$$L_3\sum_{k=1}^{d}\omega\left(\left(\sum_{i=1}^{s} x(i,k)\right) - \beta(k)\right)$$

$$L_4\sum_{\ell=1}^{s}\sum_{k=1}^{d}\omega\left(x(\ell,k) - \alpha(\ell)\right)$$

$$L_5\omega\left(\sum_{k=1}^{d}(\beta(k) - m(k))\right)$$

$$L_6\left(-\omega\left(\sum_{k=1}^{d}\beta(k)\right)\right)$$

$L_i > 0, i = 1, \cdots, 6$ are penalty parameters. The function ω is defined as follows,

$$\omega(x) = \begin{cases} x & if \quad x \geq 0 \\ 0 & if \quad x < 0 \end{cases}$$

The derivative $\omega'(x)$ is

$$\omega'(x) = \begin{cases} 1 & if \quad x > 0 \\ 0 & if \quad x < 0 \end{cases}$$

ω is not differentiable at $x = 0$. However, take the derivative at $x = 0$ to be 1. We use gradient descent method to find optimal solution of the above unconstrained minimization problem. The optimal solutions $x(\ell, k)$ are also optimal solutions to the original constrained optimization problem. We present an algorithm to find optimal solution of the unconstrained minimization problem.

Algorithm

Initialize Parameters in the model
Choose δ the step size in gradient descent
for $n = 1 : maxiter$ do
  if $\left(\sum_{i=1}^{\beta(k)} e^{-\Delta(x(\ell,k)-i)^2} - 1 - \epsilon \geq 0\right)$ $\omega_1 = 1$ else $\omega_1 = 0$ end
  if $\left(1 - \epsilon - \sum_{i=1}^{\beta(k)} e^{-\Delta(x(\ell,k)-i)^2} \geq 0\right)$ $\omega_2 = 1$ else $\omega_2 = 0$ end
  if $\left(\sum_{\ell=1}^{s}\sum_{k=1}^{d}\omega\left(x(\ell,k) - \alpha(\ell)\right) \geq 0\right)$ $\omega_4 = 1$ else $\omega_4 = 0$ end
  if $\left(\sum_{k=1}^{d}\omega\left(\left(\sum_{i=1}^{s} x(i,k)\right) - \beta(k) \geq 0\right)\right)$ $\omega_3 = 1$ else $\omega_3 = 0$ end
  if $\left(\left(\sum_{k=1}^{d}\beta(k) - m(k)\right) \geq 0\right)$ $\omega_5 = 1$ else $\omega_5 = 0$ end
  if $\left(-\left(\sum_{k=1}^{d}\beta(k)\right)\right) \geq 0$ $\omega_6 = 1$ else $\omega_6 = 0$ end
  Compute the gradient of the objective function.
  Update $x(\ell, k)$

Update the objective function

$$
\begin{aligned}
&-\sum_{\ell=1}^{s}\sum_{k=1}^{d} c(\ell,k)x(\ell,k) + \sum_{\ell=1}^{s}\sum_{k=1}^{d} b(\ell,k)x(\ell,k) \\
&+L_1\sum_{\ell=1}^{s}\sum_{k=1}^{d}\omega_1 * \left(\sum_{i=1}^{\beta(k)} e^{-\Delta(x(\ell,k)-i)^2} - 1 - \epsilon\right) \\
&+L_2\sum_{\ell=1}^{s}\sum_{k=1}^{d}\omega_2 * \left(1 - \epsilon - \sum_{i=1}^{\beta(k)} e^{-\Delta(x(\ell,k)-i)^2}\right) \\
&L_3\sum_{k=1}^{d}\omega_3 * \left(\left(\sum_{i=1}^{s} x(i,k)\right) - \beta(k)\right) \\
&L_4\sum_{\ell=1}^{s}\sum_{k=1}^{d}\omega_4 * (x(\ell,k) - \alpha(\ell)) \\
&L_5\omega_5 * \left(\sum_{k=1}^{d}(\beta(k) - m(k))\right) \\
&L_6\left(-\omega_6 * \left(\sum_{k=1}^{d}\beta(k)\right)\right)
\end{aligned}
$$

$n = n + 1$

end

3. **Numerical Examples**

In this section we provide two numerical examples. In both examples we consider two sources and two destinations. $s = 2$ and $d = 2$. The difference between the two examples is the initial approximation to the solution and the number of iterations.

Example-1

In this example, the cost of transportation of a batch from source i to destination j is given by the following matrix where c_{ij} is the cost per batch from source i to destination j. These costs are revenues for the freight transportation companies.

$$c = \begin{pmatrix} 25 & 17 \\ 18 & 17 \end{pmatrix}$$

The operational cost of transportation for the freight companies from source i to destination j is given by the following matrix where b_{ij} is the cost per batch from source i to destination j.

$$b = \begin{pmatrix} 2 & 3 \\ 1 & 2 \end{pmatrix}$$

$$\beta = [15, 9] \quad \alpha = [16, 4] \quad m = [16, 10] \quad \Delta = 14 \quad \epsilon = 0.000005$$

L_i is taken to be 10^6 for all $i = 1, \cdots, 6$. The initial approximation to the solution $x(i,j)$ is given by the following matrix. The step length parameter $\delta = 10^-6$

$$x = \begin{pmatrix} 5 & 6 \\ 2 & 3 \end{pmatrix}$$

x_{ij} is the number of batches transported from source i to destination j. A MATLAB code is written to ex-cute the algorithm and obtain the optimal number of batches from sources to destinations. The optimal solution is given by the following matrix.

$$x = \begin{pmatrix} 13.9995 & 3 \\ 5 & 2 \end{pmatrix}$$

Thus in the optimal solution, 14 batches are transported from source 1 to destination 1, 3 batches from sources 1 to destination 2, 5 batches from source 2 to destination 1, and 2 batches from source 2 to destination 2. The optimal objective function value (that is the revenue for the freight company) for the maximization problem we had at the beginning is found to be 478.9875.

Example-2

In this example we take the same values of the parameters as in the first example. The maximum number of iterations for this example is half of the number of iterations used in the first example. The initial approximation of the solution in this example is given by the following matrix.

$$x = \begin{pmatrix} 12 & 4 \\ 3 & 2 \end{pmatrix}$$

The optimal solution is found to be the same as the one obtained in the first example.

$$x = \begin{pmatrix} 13.9995 & 3 \\ 5 & 2 \end{pmatrix}$$

Thus in the optimal solution, 14 batches are transported from source 1 to destination 1, 3 batches from sources 1 to destination 2, 5 batches from source 2 to destination 1, and 2 batches from source 2 to destination 2. The optimal objective function value (that is the revenue for the freight company) for the maximization problem we had at the beginning is found to be 478.9875.
The optimal solutions and the optimal objective function values in the two examples are the same. We took two different initial approximations of the optimal solution and obtained the same optimal solution in both cases.

4. **Conclusion**

In this paper, we presented a nonlinear programming approach to determine an optimal scheduling of freight transportation from of sources to destinations. Items to be transported are grouped in batches. The decision variables are the number of batches from sources to destinations. These decision variables take on integer values. A nonlinear constraint is added to the optimization model to enforce these decision variables to be integers within a tolerance limit of ϵ (which is taken to be a small number). We transformed the constrained optimization problem to an unconstrained optimization problem using a penalty method. A gradient descent method is used to find an optimal solution of the unconstrained optimization problem. The choice of the step length in the gradient descent method depends on the norm of the gradient of the objective function. We select the an appropriate step length by trial and error. The initial approximation of the solution also plays an important role in the convergence of the algorithm to an optimal solution. There is no rule for the choice of the initial approximation of the solution but starting with some feasible solution will help. The number of iterations needed to get to n optimal solution depends on the problem and one has to make an adjustment of to the maximum number of iterations (increase or decrease) based on the outcome of a solution to the decision variables until we get integer or close to integer values within the tolerance limit.
The objective function in the maximization problem is linear and there could be more than one possible solution to the problem. In the optimal scheduling problem, there could be a number of sources, a number of destinations, many batches of items to be transported from sources to destinations. A single carrier (truck company) may not be able to perform all the task. Once the optimal scheduling is determined, different carriers can cooperate and share revenue based on the task each carrier performed. For example, a Shapley value can be used to divide the revenue among the carriers who cooperated to perform the task.

5. Acknowledgements

This research is supported by the Center for Connected Multimodal Mobility (C^2M^2) (USDOT Tier 1 University Transportation Center) headquartered at Clemson university, It is also partially supported by the National Science Foundation Grant No. 1954532.

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