

PORTFOLIO SELECTION PROBLEM

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ABSTRACT.

Given a set of securities or assets it is of interest to find an optimal way of investing in these assets. What is optimal has to be specified. The objective is to optimize the return consistent with the specified objective. When there are several assets it is unlikely all the assets will increase if they are correlated. It is necessary to diversify one's assets for a secure return. To deal with the different assets a combination of the assets should be considered with constraints as needed. One approach is the Markowitz mean-variance model where the mean variance is minimized including constraints. In this paper neural networks and machine learning are used to extend the ways of dealing with portfolio asset allocation.

Portfolio selection problem in an efficient way. The use of heuristic algorithms in this case is imperative. In the past some heuristic methods based mainly on evolutionary algorithms, tabu search and simulated annealing have been developed. The purpose of this paper is to consider a particular neural network model, the Hopfield network, which has been used to solve some other optimisation problems and apply it here to the portfolio selection problem, comparing the new results to those obtained with previous heuristic algorithms. Although great success has been achieved for portfolio analysis with the birth of Markowitz model, the demand for timely decision making has significantly increased especially in recent years with the advancement of high frequency trading (HFT), which combines powerful computing servers and the fastest Internet connection to trade at extremely high speeds. This demand poses new challenges to portfolio solvers for real-time processing in the face of time-varying parameters. Neural networks, as one of the most powerful machine learning tools has seen great progress in recent years for financial data analysis and signal processing ([1], [14]). Using computational methods, e.g., machine learning and data analytics, to empower conventional finance is becoming a trend widely adopted in leading investment companies ([3]).

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1. Introduction

In a portfolio selection problem there are certain securities or assets of interest and the objective is to decide how to invest in each of these assets for the best return. Since all the assets do not necessarily give a positive return one has to diversify the total amount of assets in the assets for the best portfolio return. A possible strategy for choosing a portfolio selection problem is one due to Markowitz ([22]) based on mean and variance of the assets in the portfolio where the variance is to be minimized and the return is maximized leading to a multi-objective optimization problem.

There are some methods that use evolutionary algorithms([21],[27]), ([9],[25]).

Although great success has been achieved for portfolio analysis with the birth of Markowitz model, the demand for timely decision making has significantly increased especially in recent years with the advancement of high frequency trading (HFT), which combines powerful computing servers and the fastest Internet connection to trade at extremely high speeds. This demand poses new challenges to portfolio solvers for real-time processing in the face of time-varying parameters. Neural networks, as one of the most powerful machine learning tools (Khan et al., 2020, 2022), has seen great progress in recent years for financial data analysis and signal processing ([1]). Using computational methods, e.g., machine learning and data analytics, to empower conventional finance is becoming a trend widely adopted in leading investment companies ([3] , [26]).

In this paper we use neural network and machine learning approach to forecast possible returns as well as solve the constrained multi-objective problems.

2. Portfolio selection

Let us consider 5 assets with daily open and closing prices (4/12/24 back to 3/14/24, Nasdaq), $R_{i,t}, i = 1, 2, 3, 4, 5 t = 1, 2, 3, \dots, 21$. Let $r_{it}, i = 1, 2, 3, 4, 5; t = 1, 2, 3, \dots, 20$ the rate of returns, μ_i the geometric mean of asset i , $\sigma_{ij}, i, j = 1, 2, \dots, 5$ the covariance between assets i and j . We have

$$\begin{aligned}
 r_{it} &= \frac{R_{i,t} - R_{i,t-1}}{R_{i,t-1}}, \\
 \mu(i) &= \left[\prod_{t=1}^{20} (1 + r_{it}) \right]^{1/20} - 1, \\
 \Sigma_{ij} &= \frac{1}{20} \sum_{t=1}^{20} (r_{it} - \hat{r}_i)(r_{jt} - \hat{r}_j),
 \end{aligned}
 \tag{2.1}$$

where \hat{r}_i, \hat{r}_j are the arithmetic mean of assets i and j . Let us give a general description

of the Markowitz mean-variance model. Let N be the number of assets and μ_i the mean return of asset i and σ_{ij} be the covariance of assets i and j . Let x_i , be the fraction of the total that is to be invested in asset i . We would like to minimize

$$\sum_{j=1}^N \sum_{i=1}^N \rho_{ij} \sigma_i \sigma_j x_i x_j$$

and maximize

$$\sum_{i=1}^N x_i \mu_i,$$

where $\rho_{ij} \sigma_i \sigma_j$ is the covariance of the securities i and j . Thus we have a multi-objective optimization problem. A possible minimizer is $\{0.5018, 0.0405, 0.0019, 0.0273, 0.4285\}$ after solving the multi-objective problem. $\{0.1971, 0.1982, 0.1986, 0.1979, 0.1995\}$ minimizer based on original 21 values for each asset using neural networks approach $\{0.1985, 0.1986, 0.1974, 0.1981, 0.1988\}$ minimizer values based on predicted 10 values for each asset using neural networks. We note from the figure for mean variance that it decreases to zero.

A multi-objective optimisation problem has several different optimal solutions that are non-dominated. One can choose one of them based on additional decision making criterion or criteria. Later we will solve a multi-objective problem resulting from a concrete set of 5 assets and 21 opening and closing prices mentioned in the beginning of this section. We will use neural networks methodology as part of our methodology

In the following tables we present our data for opening and closing stock prices.

Next we proceed to solve various formulations of the multi-objective problem. Asset allocations may be made using known data and forecasting what could be in the future. Toward this objective we introduce a forecasting procedure.

3. Forecasting Using Grey System

We use the assets, returns, mean and variance to predict future mean returns and covariances. We verify the newly generated returns are acceptable using SVM (Support Vector Machine) machine learning algorithm. We now present the traditional forecasting model GM(1,1). The traditional forecasting model $GM(1,1)$ is as follows. We start with a nonnegative sequence

$$X^0 = \{x^0(1), x^0(2), x^0(3), \dots, x^0(n)\}$$

of raw data and its accumulating generation (*AGO*) sequence

$$X^1 = \{x^1(1), x^1(2), x^1(3), \dots, x^1(n)\}$$

TABLE 1. Apple Prices

Opening Price	Closing Price
174.2600	176.5500
168.3400	175.0400
168.8000	167.7800
168.7000	169.6700
169.0300	168.4500
169.5900	169.5800
170.2900	168.8200
168.7900	169.6500
169.0800	168.8400
171.1900	170.0300
171.7500	171.4800
170.4100	173.3100
170.0000	169.7100
170.5700	170.8500
171.7600	172.2800
177.0500	171.3700
175.7200	178.6700
174.3400	176.0800
175.5700	173.7200
171.1700	172.6200
172.9100	173.0000

where $x^1(1) = x^0(1)$, and

$$x^1(k) = \sum_1^k x^{(0)}(i), \quad k = 1, 2, \dots, n$$

and the mean sequence generated consecutive neighbours of X^1 is

$$Z^1 = \{z^1(1), z^1(2), z^1(3), \dots, z^1(n)\}$$

where

$$z^1(k) = (x^1(k) + x^1(k-1))/2, \quad k = 2, \dots, n.$$

The equation

$$x^0(k) + az^1(k) = b$$

TABLE 2. Cisco Prices

Opening Price	Closing Price
49.2100	48.4900
49.3200	49.5300
49.7300	49.2100
48.8400	50.0100
48.1900	48.2400
47.9600	48.5500
49.0900	48.1100
48.8000	48.7400
49.9600	49.3600
49.9600	50.0400
49.8900	49.9100
49.7700	49.7700
49.5500	49.5500
49.5600	49.6800
50.0500	49.7800
49.7100	49.8800
49.0400	49.5500
49.1200	49.1400
49.2200	49.2700
49.7900	48.9300
50.2700	49.7900

is called basic form of the GM(1,1) model and the whitenization equation is established

$$dx^{(1)}/dt + ax^{(1)} = b$$

The whitenization equation is solved and the prediction values of $X(1)$ can be calculated as follows.

$$x^{(1)}(k) = (x^{(0)}(1) - b/a)e^{-a(k-1)} + b/a, k = 2, 3, \dots$$

Therefore the predicted values can be generated by

$$x^{(0)}(1) = x^{(0)}(1),$$

TABLE 3. Meta, Inc. Prices

Opening Price	Closing Price
517.7500	511.9000
521.1100	523.1600
509.2900	519.8300
522.2300	516.9000
529.2800	519.2500
516.8600	527.3400
516.4200	510.9200
498.9300	506.7400
485.1000	497.3700
487.2000	491.3500
492.8400	485.5800
499.3000	493.8600
505.1300	495.8900
505.7900	503.0200
507.0000	509.5800
514.7100	507.7600
499.5000	505.5200
488.1700	496.2400
491.9100	496.9800
489.0100	484.1000
500.2600	491.8300

$$x^{(0)}(k) = x^{(1)}(k) - x^{(1)}(k-1), k = 2, 3, \dots, n.$$

Relative percentage error (RPE) and mean absolute percentage error (MAPE) are used to evaluate the overall forecast performance accuracy of the prediction models. They are defined as follows:

$$RPE(k) = \left| \frac{x^{(0)}(k) - x^{(0)}(1)}{x^{(0)}(k)} \right| \times 100 \text{ } 0/0$$

$$MAPE = 1/n \sum_{k=1}^n RPE(k).$$

. The REP (Relative error percentage) between the predicted 10 cases of predicted and the actual 11th to the 21 th actual data can be calculated. The mean

TABLE 4. Microsoft Prices

Opening Price	Closing Price
424.0500	421.9000
425.8200	427.9300
422.1900	423.2600
426.4400	426.2800
425.1700	424.5900
420.0100	425.5200
424.9900	417.8800
419.7300	420.4500
420.1100	421.4400
423.9500	424.5700
420.9600	420.7200
424.4400	421.4300
425.6100	421.6500
425.2400	422.8600
429.7000	428.7400
429.8300	429.3700
422.0000	425.2300
417.8300	421.4100
414.2500	417.3200
419.2900	416.4200
420.2400	425.2200

absolute percentage error is .42.

Next we will present our forecasting of the assets based on the ones we have in the tables above. Using SVM machine learning procedure we test the ACCEPTABILITY OF THE PREDICTED VALUES.

4. Support Vector Machines Analysis in Portfolio Selection

In this section we consider SVM approach to judge the acceptability of the forecast portfolio values. We consider the following constrained optimization problem.

$$\min \left\{ \frac{1}{2} \sum_{i=1}^L W_i^2 + \gamma \sum_{i=1}^k e_i^2 \right\}$$

subject to

TABLE 5. StarBucks Prices

Opening Price	Closing Price
85.4200	84.9200
86.0000	85.9000
86.3200	85.9500
87.1900	87.1300
86.9700	87.1900
87.9000	86.8500
89.1300	87.6200
89.6000	88.7000
91.1100	89.3000
91.8200	91.5300
91.7900	91.3900
90.6500	91.5000
90.9800	90.3600
90.6500	90.6700
91.7400	90.7100
92.6300	91.6600
91.7900	92.6000
91.0000	91.5900
90.7500	91.0100
91.6000	90.1200
91.1000	91.6600

TABLE 6. Predicted Apple Prices

Opening Price	Closing Price
171.7500	171.4800
170.3358	171.0123
170.5143	171.1617
170.6930	171.3112
170.8719	171.4609
171.0510	171.6107
171.2303	171.7606
171.4097	171.9107
171.5894	172.0609
171.7692	172.2112

TABLE 7. Predicted Cisco Prices

Opening Price	Closing Price
49.8900	49.9100
49.7860	49.9919
49.7813	49.9828
49.7766	49.9736
49.7720	49.9645
49.7673	49.9554
49.7626	49.9463
49.7580	49.9371
49.7533	49.9280
49.7486	49.9189

TABLE 8. Predicted Meta Prices

Opening Price	Closing Price
492.8400	485.5800
508.2692	502.7410
506.1967	501.8563
504.1326	500.9731
502.0770	500.0915
500.0297	499.2115
497.9908	498.3330
495.9602	497.4560
493.9378	496.5806
491.9238	495.7068

$$y_l = \sum_{i=1}^L W_i \phi(x_{il}) + b + e_l, \quad l = 1, 2, \dots, k.$$

We choose W_i , b , e_k to decide if a prediction is acceptable.

We will consider the following optimization problem

$$\min \left\{ \frac{1}{2} \sum_{i=1}^{10} W_j^2 + \gamma \sum_{i=1}^{10} e_i^2 \right\}$$

subject to

TABLE 9. Predicted Microsoft Prices,

Opening Price	Closing Price
420.9600	420.7200
428.2753	425.3303
426.9804	424.6740
425.6894	424.0187
424.4023	423.3643
423.1191	422.7110
421.8398	422.0588
420.5643	421.4075
419.2927	420.7572
418.0250	420.1079

TABLE 10. Predicted Star Bucks Prices

Opening Price	Closing Price
91.7900	91.3900
91.0537	91.1249
91.1176	91.1276
91.1817	91.1302
91.2457	91.1329
91.3098	91.1356
91.3740	91.1382
91.4382	91.1409
91.5025	91.1435
91.5667	91.1462

$$y_1 = 1/2 \sum_{j=1}^{10} W_j R1(j, 2) + b + e_1,$$

Let

$$T1 = y_1 - 1/2 \sum_{j=1}^{10} W_j R1(j, 2) + b + e_1.$$

Net, consider

$$\min \left\{ \frac{1}{2} \sum_{i=1}^{10} W_j^2 + \gamma \sum_{i=1}^{10} e_i^2 \right\}$$

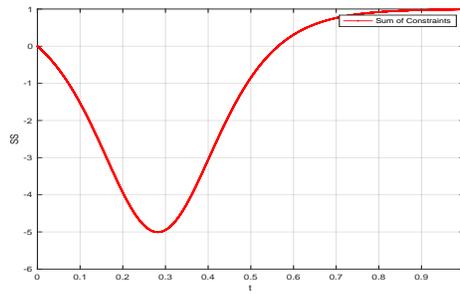


FIGURE 1. Sum-Of-Assignments

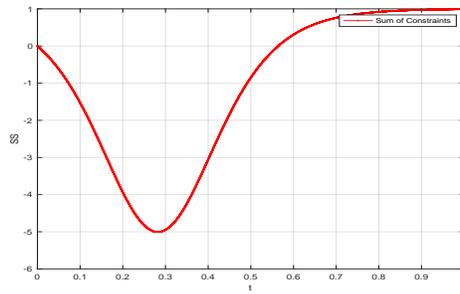


FIGURE 2. Sum-Of-Assignments-Based on Predicted Values

subject to

$$y_2 = 1/2 \sum_{j=1}^{10} W_j R_2(j, 2) + b + e_2.$$

Let

$$T_2 = y_2 - 1/2 \sum_{j=1}^{10} W_j R_2(j, 2) + b + e_2$$

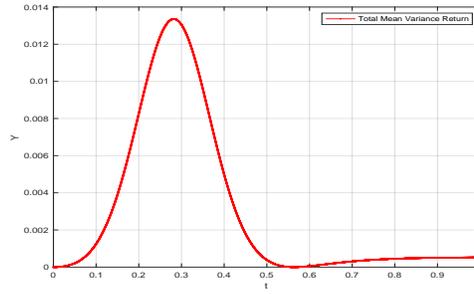


FIGURE 3. MeanVarianceSum

$$\min\left\{\frac{1}{2} \sum_{i=1}^{10} W_j^2 + \gamma \sum_{i=1}^{10} e_i^2\right\}$$

subject to

$$y_1 = 1/2 \sum_{j=1}^{10} W_j R3(j, 2) + b + e_1.$$

Let

$$T3 = y_3 - 1/2 \sum_{j=1}^{10} W_j R3(j, 2) + b + e_3.$$

$$\min\left\{\frac{1}{2} \sum_{i=1}^{10} W_j^2 + \gamma \sum_{i=1}^{10} e_i^2\right\}$$

subject to

$$y_4 = 1/2 \sum_{j=1}^{10} W_j R4(j, 2) + b + e_4.$$

Let

$$T4 = y_4 - 1/2 \sum_{j=1}^{10} W_j R4(j, 2) + b + e_4.$$

TABLE 11. (T1, PT1),..., (T5,PT5)

T1= -0.0019	PT1= -0.0019	T1-PT1=2.4063e-05
T2= -0.0005	PT2= -0.0006	T2-PT2= 2.0074e-05
T3= -0.0057	PT3= -0.0055	T3-PT3=-2.0820e-04
T4= -0.0047	PT4= -0.0050	T4-PT4=2.6063e-04
T5= -0.0010	PT5= -0.0011	T5-PT5=1.0000e-04

$$\min\left\{\frac{1}{2} \sum_{i=1}^{10} W_j^2 + \gamma \sum_{i=1}^{10} e_i^2\right\}$$

subject to

$$y_5 = 1/2 \sum_{j=1}^{10} W_j R5(j, 2) + b + e_5.$$

Let

$$T5 = y_5 - 1/2 \sum_{j=1}^{10} W_j R5(j, 2) + b + e_5.$$

We replace $T1, T2, T3, T4, T5$ by $PT1, PT2, PT3, PT4, PT5$ when we replace $R1(j, 2), R2(j, 2), R3(j, 2), R4(j, 2), R5(j, 2)$ by $PR1(j, 2), PR2(j, 2), PR3(j, 2), PR4(j, 2), PR5(j, 2)$. In Table 11 the corresponding values of $T1, \dots, T5$ and $PT1, \dots, PT5$ are shown from which we conclude the predictions are pretty good.

5. SVM for Forecasting and Classification

In this section we use SVM to forecast and possible daily change.

The main idea of SVMs is to construct a hyperplane as the decision surface so that the margin of separation between positive and negative examples is maximised ([28]).

Given a training set of instance-label pairs $(x_{il}, y_l), i = 1, \dots, L; l = 1, \dots, k$ and $y_i \in \{-1, 1\}$, indicating y_i the class to which the point x_{il} belongs, the SVMs require the solution of the following problem:

$$\min \left\{ \frac{1}{2} \sum_{i=1}^L W_i^2 + \gamma \sum_{i=1}^k e_i^2 \right\} + C \sum_{i=1}^k \xi_i$$

subject to

$$y_l \cdot \left(\sum_{i=1}^L W_i \phi(x_{il}) + b + e_l \right) \geq 1 - \xi_l, \quad l = 1, 2, \dots, k, \quad \xi_l \geq 0.$$

Here ξ_i are the slack variables introduced by the method which measures the degree of misclassification of the data $\{x_{il}\}$; W is the normal vector to the hyperplane. The constants are chosen to measure offsets from the hyperplane chosen as in consideration of the Grey System, and $C \geq 0$ is a chosen penalty parameter controlling the error term so that it would not be too large. We can use the SVM formulation be used for classification or forecasting as was done in the previous section. We note that $l, l = 1, \dots, k$ indicate that we are classifying data of the stock prices coming from k stocks. SVM are used for classification and prediction ([7], [8], [19], [15], [16]).

6. Conclusions

In this work we have focused on solving the portfolio selection problem and tracing out its efficient frontier. Instead of using the standard Markowitz mean-variance model, we have used a generalization of it that includes cardinality and bounding constraints. Dealing with this kind of constraints, the portfolio selection problem becomes a mixed quadratic and integer programming problem for which no computational efficient algorithms are known. We have developed a heuristic method based on the Hopfield neural network and we have used it to solve the general mean-variance portfolio selection model. The results obtained have been compared to those obtained using three other heuristic methods coming from the fields of genetic algorithms, tabu search and simulated annealing. All the experimental results presented in this paper lead us to conclude that none of the four heuristic methods has outperformed the others in all the comparisons considered. Anyway, we must specially mention the fact that the neural network model has given us a set of solutions with higher quality than the solutions from the other three heuristic methods, although they did not outstand in their number.

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REFERENCES

- [1] ADAMU, J. A., Advanced stochastic optimization algorithm for deep learning artificial neural networks in banking and finance industries. *Risk and Financial Management*,1(1),8–28 (2019).
- [2] D. ALDILA, T. GOTZ, E. SOEWONO, An optimal Control problem arising from a dengue disease transmission model, *Math. Biosci.* 242 (1)(2013), 9-16.
- [3] ALLEN,F., GU,X., & JAGTIANI, J., Asurvey of intech research and policy discussion, *Review of Corporate Finance*, 1, 259–339, 2021.
- [4] JYOTIRMAYEE BEHERA, AJIT KUMAR PASAYAT, HAREKRUSHNA BEHERA, PANKAJ KUMAR, Prediction based mean-value-at-risk portfolio optimization using machine learning regression algorithms for multi-national stock markets, *Engineering Applications of Artificial Intelligence* 120(2023)105843.
- [5] BJÖRK,T., MURGOCI, A., ZHOU, X.Y., Mean–variance portfolio optimization with state-dependent risk aversion, *Math. Finance: Int. J. Math. Stat. Finance. Econ.* 24(1),1–24, 2014.
- [6] CHANG T-J, MEADE N, BEASLEY J, SHARAIHA Y, Heuristics for cardinality constrained portfolio optimisation. *Computers & Operations Research* 2000;27:1271-1302.
- [7] CAO, L. AND TAY, Financial forecasting using support vector machines, *Neural Computing Applications*, 10, 184-192, 2001.
- [8] CAO, L. AND TAY, F.,Support vector machine with adaptive parameters in financial time series forecasting, *IEEE Transactions on Neural Networks*, 14, 1506-1518, 2003
- [9] CHANG, T.-J., MEADE, N., BEASLEY, J. E., AND SHARAIHA, Y. M, Heuristics for cardinality constrained portfolio optimisation, *Computers & Operations Research*, 27(13),1271–1302, 2000.
- [10] DAVIS, M. H., AND NORMAN, A. R., Portfolio selection with transaction costs, *Mathematicsof Operations Research*,15(4),676–713. 1990.
- [11] DRUCKER, H., BURGESS, C. J., KAUFMAN, L., SMOLA, A., VAPNIK,V., Support vector regression machines, *Adv. Neural Inf. Process. Syst.* 9, 1996.
- [12] ALBERTO FERNANDEZ, GOMEZ, S., Portfolio selection using neural networks, Alberto Fernández, Sergio Gómez, *Computers & Operations Research* 34 (2007) 1177-1191.
- [13] FIELDSEND J, MATATKO J, PENG M, Cardinality constrained portfolio optimisation.
- [14] GAO, W., & SU, C., Analysis on block chain financial transaction under artificial neural network of deep learning, *Journal of Computational and Applied Mathematics*, 380,112–131, 2020.
- [15] Z GAO, SC FANG, J LUO, N MEDHIN, A kernel-free double well potential support vector machine with applications *European Journal of Operational Research* 290 (1), 248-262, 2022.
- [16] Z GAO, SC FANG, X GAO, J LUO, N MEDHIN, A novel kernel-free least squares twin support vector machine for fast and accurate multi-class classification, *Knowledge-Based Systems* 226, 107123, 2021.
- [17] HOPFIELD J, Neural networks and physical systems with emergent collective computational abilities. *Proceedings of the National Academy of Sciences* 1982; 79:2554-2558.
- [18] PETERSON, C., SÖDERBERG, B., A New Method for Mapping Optimization Problems onto Neural Networks. *International Journal of Neural Systems* 1989;1:3-22.
- [19] LEE, M.-C., Using support vector machine with a hybrid feature selection method to the stock trend prediction, *Expert Systems with Applications*, 36 (8), 10896–10904, 2009
- [20] LIN D, WANG S, YAN H, A multiobjective genetic algorithm for portfolio selection problem. In: *Proceedings of ICOTA 2001*. Hong Kong, December 15-17, 2001.12

- [21] LIN, G., LIN, A., GU, D., Using support vector regression and K-nearest neighbors for short-term traffic flow prediction based on maximal information coefficient, *Inform. Sci.* 608, 517–531, 2022.
- [22] MARKOWITZ H, Portfolio selection. *Journal of Finance* 1952;7:77-91.
- [23] MIETTINEN, K., *Nonlinear multiobjective optimization*, Springer Science & Business Media, 2012
- [24] RAEI,R., AND ALIBEIKI, H., Portfolio optimization using particle swarm optimization method, *Financial Research Journal*,12(29),121–149, 2010.
- [25] SCHAERF A, Local Search Techniques for Constrained Portfolio Selection Problems. *Computational Economics* 2002;20:177-190.
- [26] TSANTEKIDIS, A., PASSALIS, N., TOUFA, A-S., SAITAS-ZARKIAS, K., CHAIRISTANIDIS, S., AND TEFAS,A., Price trailing for financial trading using deep reinforcement learning, *IEEE Transactions on Neural Networks and Learning Systems*, 32(7), 2837–2846, 2020.
- [27] XIA Y, LIU B, WANG S, LAI K, A model for portfolio selection with order of expected returns. *Computers & Operations Research* 2000;27:409-422.
- [28] XU, X., ZHOU, C. AND WANG, Z.,Credit scoring algorithm based on link analysis ranking with support vector machine. *Expert Systems with Applications*, 36, 2625–2632, 2009.
- [29] YAMAN, I., AND DALKILIC, T., Ahybrid approach to cardinality constraint portfolio selection problem based on nonlinear neural network and genetic algorithm. *Expert Systems with Applications*,169,401–517, 2021.